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GUIDANCE NOTES ON

SPECTRAL-BASED FATIGUE ANALYSIS FOR VESSELS

(FOR THE 'SFA (*years*)' CLASSIFICATION NOTATION)

JANUARY 2004 (Updated February 2012 – see next page)

Updates

February 2012 consolidation includes:

- November 2009 version plus Notice No. 4 and Corrigenda/Editorials.

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- August 2006 – Notice No. 1
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Foreword

This Guide provides information about the optional classification notation, 'Spectral Fatigue Analysis' – **SFA (*years*)** – which is available to qualifying vessels as described in 1-1-3/20 of the ABS *Rules for Building and Classing Steel Vessels*, referred to herein as the *Steel Vessel Rules*.

This guidance document is referred to herein as "this Guide" and its issue date is January 2004. Users of this Guide are encouraged to contact ABS with any questions or comments concerning this Guide. Users are advised to check with ABS to ensure that this version of the Guide is current.

GUIDANCE NOTES ON

SPECTRAL-BASED FATIGUE ANALYSIS FOR VESSELS

CONTENTS

SECTION 1 Introduction

1 Purpose and Applicability *(1 December 2007)*

Part 5C of the ABS *Rules for Building and Classing Steel Vessels (Steel Vessel Rules)* presents the simplified fatigue assessment criteria for the classification of the various types of specialized vessels covered by the Rules. A brief description of the background and objectives for these fatigue criteria is given in Subsection 1/3, below.

In addition to the simplified fatigue strength criteria for ABS's classification purposes, the Owner may wish to apply more extensive Spectral-based Fatigue Analysis (SFA) techniques to the vessel's structural systems. It may be an added objective of these Spectral-based Fatigue Analyses to demonstrate longer target fatigue lives than those minimally required for the classification of a vessel.

The term, "*more extensively*," means that the Spectral-based Fatigue Analysis technique, rather than the SafeHull Fatigue Assessment technique, a Permissible Stress Range method (see Subsection 1/3 below), is used to verify the adequacy of the fatigue lives of the critical locations in the structural system. The SafeHull Fatigue Assessment technique is still to be employed in the overall design and analysis effort for the structure. The results of the fatigue assessment should be used in the selection of areas for which the Spectral-based Fatigue Analyses will be done.

ABS is to be consulted and is to agree on the structural locations that are to be subjected to the Spectral-based Fatigue Analysis.

In recognition of the appropriate, additional use of Spectral-based Fatigue Analysis, ABS will grant the optional classification notation, **SFA (*year*).** The **SFA (*year*)** notation means that the design fatigue life value is equal to 20 years or greater. The value in the parentheses is the design fatigue life equal to 20 years or more (in 5-year increments), as specified by the applicant. It should be understood that only one, minimum designated life value is applied to the entire structural system. The fatigue life value refers to the target value set by the designer and not the value calculated in the analysis. The calculated values are usually much higher than the target values specified for design.

For vessels complying with Part 5A "Common Structural Rules for Double Hull Tankers" or Part 5B "Common Structural Rules for or Bulk Carriers" of the ABS *Rules for Building and Classing Steel Vessels*, the design target fatigue life for Spectral-based Fatigue Analysis is equal to 25 years.

3 Background

In the *Steel Vessel Rules* applied to a vessel, design and analysis for fatigue strength is usually accomplished through a combination of methods. Designers commonly make primary use of what is referred to as the SafeHull "Fatigue Assessment" technique (e.g., for an oil tanker with length greater than 150 m: *Steel Vessel Rules* Appendix 5C-1-A1). This is a "designer-oriented", permissible stress range approach that is readily applied to a large portion of the fatigue-critical structural details of a vessel's hull structure. This technique was derived by considering "unrestricted ocean service" environmental loadings (due to waves) and a design target fatigue life of 20 years.

The *Steel Vessel Rules* do not preclude the imposition of requirements by ABS to demonstrate the adequacy of fatigue strength of structural components by additional or other techniques that include the Spectral-based Fatigue Analysis methods. Indeed, it is commonly necessary to perform Spectral-based Fatigue Analysis of structural details, which is beyond the range of applicability of the permissible stress range fatigue assessment approach. Furthermore, the vessel owner or designer is free to increase the target fatigue lives of some or all of the structural components above the 20-year minimum value, which is recognized by the optional classification notation, **FL (*years*)**, in the *Steel Vessel Rules* and other ABS classification standards. Therefore, the incidental or supplementary use of Spectral-based Fatigue Analysis methods is not a reason to grant the **SFA (*years*)** notation.

5 Areas for Fatigue Strength Evaluation

Reference should be made to the *Steel Vessel Rules* for specific "Guidance on Locations" that should be included in the fatigue assessment.

7 Detailed Contents of this Guide

Spectral-based Fatigue Analysis is a complex and numerically-intensive technique. As such, there is more than one variant of the method that can be validly applied in a particular case. ABS does not wish to preclude the use of any valid variant of a Spectral-based Fatigue Analysis method by "over specifying" the elements of an approach. However, there is a need to be clear about the basic minimum assumptions that are to be the basis of the method employed, and some of the key details that are to be incorporated in the method to produce results that will be acceptable to ABS. For this reason, most of the remainder of this Guide is a presentation on these topics.

As for the main assumptions underlying the Spectral-Based Fatigue Analysis method, these are as follows.

i)   Ocean waves are the source of the fatigue inducing stress range acting on the structural system being analyzed.

ii)   In order for the frequency domain formulation and the associated probabilistically based analysis to be valid, load analysis and the associated structural analysis are assumed to be linear. Hence, scaling and superposition of stress range transfer functions from unit amplitude waves are considered valid.

iii)   Non-linearities, brought about by non-linear roll motions and intermittent application of loads such as wetting of the side shell in the splash zone, are treated by correction factors.

iv)   Structural dynamic amplification, transient loads and effects such as springing are insignificant in the typical case, hence, use of quasi-static finite element analysis is valid, and the fatigue inducing stress variations due to these types of load effects can be ignored.

Also, for the particular method presented in Appendix 3, it is assumed that the short-term stress variation in a given sea-state is a random narrow-banded stationary process. Therefore, the short-term distribution of stress range can be represented by a Rayleigh distribution.

The key components of the Spectral-based Fatigue Analysis method for the selected structural locations can be categorized into the following components:

- Establish fatigue demand

- Determine fatigue strength or capacity

- Calculate fatigue damage or expected life

These analysis components can be expanded into additional topics, as follows, which become the subject of particular Sections in the remainder of this Guide.

The topic, "Establish Fatigue Demand", is covered in Sections 2 through 7. The topics of "Determine Fatigue Strength or Capacity" and "Calculate Fatigue Damage or Expected Life" are the subjects of Sections 8 and 9, respectively. Reference can be made to Section 1, Figure 1 for a schematic representation of the Spectral-based Fatigue Analysis Procedure.

A purposeful effort is made in this Guide to avoid complicated formulations, which will detract from the concepts being presented. The most complex formulations are those relating to the calculation of fatigue damage resulting from the predicted stress range. These formulations are presented in Appendix 3 of this Guide. It is often at this formulation level that valid variations of a method may be introduced, and for that reason, it is emphasized that the contents of Appendix 3 are provided primarily to illustrate principle, rather than as mandatory parts of the Spectral-based Fatigue method.

FIGURE 1
Schematic Spectral-based Fatigue Analysis Procedure

(ForEachLocationorStructuralDetail)

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┌─────────────────────┐                          ┌─────────────────────┐
│ Establish Fatigue   │                          │    Establish        │
│ Demand              │                          │ Fatigue Strength    │
│ Sections 2 thru 7   │                          │ Section 8           │
└─────────────────────┘                          └─────────────────────┘
          │                                                  │
          ▼                                                  │
┌─────────────────────┐                                      │
│ Obtain              │                                      │
│ Environmental Data  │                                      │
│ Section 3           │                                      │
└─────────────────────┘                                      │
          │                                                  │
          ├──────────────────────────┐                       │
          ▼                          ▼                       │
┌─────────────────────┐   ┌─────────────────────┐            │
│ Perform Analysis of │   │ Determine Still-water│           │
│ Vessel Motion and   │   │ Loads and Check     │            │
│ Wave-Induced Loads  │   │ Equilibrium         │            │
│ Section 4           │   │ See 4/3             │            │
└─────────────────────┘   └─────────────────────┘            │
          │                          │                       │
          ▼                          │                       │
┌─────────────────────┐              │                       │
│ For each Heading     │             ▼                       │
│ Angle and Wave Freq. │   ┌─────────────────────┐           │
│ See 2/3              │   │ Do for each         │           │
│                      │   │ Base Vessel         │           │
│ Calculate RAOs for - │   │ Load Condition      │           │
│ • External Hydrodynamic│  │ See 2/5            │           │
│   Pressure           │   └─────────────────────┘           │
│ • Internal Tank and  │             │                       │
│   Hold Loads         │             │                       │
│ • Accelerations of   │             │                       │
│   Discrete Masses    │             │                       │
│ Section 5            │             │                       │
└─────────────────────┘             │                       │
          │◄──────────────────────────                       │
          ▼                                                  │
┌─────────────────────┐                                      │
│ Assemble Load Cases │                                       │
│ for Structural      │                                       │
│ Analysis and Check  │                                       │
│ Dynamic Equilibrium │                                       │
│ Section 6           │                                       │
└─────────────────────┘                                      │
          │                                                  │
          ▼                                                  │
┌─────────────────────┐                                      │
│ Perform Structural  │                                       │
│ Analysis to Obtain  │                                       │
│ Stress Range        │                                       │
│ Transfer Function   │                                       │
│ Section 7           │                                       │
└─────────────────────┘                                      │
          │                                                  │
          ▼                                                  │
┌─────────────────────┐                                      │
│ Calculate Fatigue   │◄──────────────┐                      │
│ Damage              │               │                      │
│ Section 9           │               │                      │
└─────────────────────┘               │                      │
          │                            │                      ▼
          ▼                            │            ┌─────────────────────┐
┌─────────────────────┐   ┌──────────────────┐     │ Apply               │
│ Calculate combined  │   │ COMPARE          │     │ Required Safety     │
│ Fatigue Damage from │──▶│ Expected Strength│◄────│ Factors             │
│ Multiple Base Vessel│   │ To Be Greater    │     └─────────────────────┘
│ Load Cases          │   │ Than or Equal to │
│ See 2/7             │   │ Expected Damage  │
└─────────────────────┘   └──────────────────┘
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SECTION 2 Establishing Fatigue Demand

1 Introduction

Sections 2 through 7 address the procedures used to estimate the fatigue demand at a structural location that is the object of the fatigue strength evaluation.

3 Stress Range Transfer Function *(1 December 2007)*

With ocean waves considered the main source of fatigue demand, the fundamental task of a spectral fatigue analysis is the determination of the stress range transfer function, $H_\sigma(\omega|\theta)$, which expresses the relationship between the stress at a particular structural location and wave frequency (ω) and wave heading (θ).

It is preferred that a structural analysis be carried out at each frequency, heading angle and "Base Vessel Loading Condition" (see Subsection 2/5) employed in the spectral analysis and that the resulting stresses are used to directly generate the stress transfer function.

Normally, the frequency range to be used is 0.2 to 1.80 radians/second in increments not larger than 0.1 rad/s. However, depending on the characteristics of the response, it may be necessary to consider a different frequency range. The wave heading range is 0 to 360 degrees in increments not larger than 30 degrees.

In the course of seakeeping analysis, 75% of vessel's design speed is to be used for the prediction of motions and pressures.

5 Base Vessel Loading Conditions

The Base Loading Conditions relate to the probable variations in loading that the hull structure will experience during its service life. The main parameters defining a Base Loading Condition are: tank or hold loading and ballast arrangements, and hull draft and trim. These parameters have a direct influence on the "static" stress components of the hull's response, but they also affect the wave-induced variable stress range experienced at a structural location. There are two direct ways that this influence is felt. First, this influence is felt in the magnitudes and distributions of masses and restoring forces in the determination of global and local accelerations and rigid body displacements, which in turn affect the wave-induced load effects employed in the structural analysis. Secondly, the variation of draft affects the areas of the hull that will be subjected to direct external pressures, and the magnitude and distribution of these pressures.

7   Combined Fatigue from Multiple Base Vessel Loading Conditions
(1 February 2012)

Because of the variability in Base Vessel Loading Conditions and its effects on the fatigue strength predictions, it is necessary to consider more than one base case in the fatigue analysis. As a minimum, two cases should be modeled and used in the Spectral-based Fatigue Analysis process. The two cases are ones resulting from, and representing, the probable deepest and shallowest drafts, respectively, that the vessel is expected to experience during its service life.

A factor of 0.85 is to be applied to account for non-sailing time for operations such as loading and unloading, repairs, etc. for conventional trading vessels. For vessels engaged in a service with less sailing time, special consideration may be given to accepting a lower factor, subject to Owner agreement.

Note: Suggested Approach: In some (so-called "Closed Form") formulations to calculate fatigue demand, the fraction of the total time for each Base Vessel Loading Condition is used directly. In this case, potentially useful information about the separate fatigue damage from each vessel loading condition is not obtained. Therefore, it is suggested that the fatigue damage from each vessel loading condition be calculated separately. The 'combined fatigue life' is then calculated as a weighted average of the reciprocals of the lives resulting from considering each case separately. For example, if two base loading conditions are employed, and the calculated fatigue life for a structural location due to the respective base vessel loading conditions are denoted L_1 and L_2, and it is assumed that each case is experienced for one-half of the sailing time during the vessel's service life, then the combined fatigue life, L_C, is:

$$L_C = 1/0.85[0.5(1/L_1) + 0.5(1/L_2)].$$

As a further example, if there were three base vessel loading conditions L_1, L_2, and L_3 with exposure time factors of 40, 40, and 20 percent, respectively; then the combined fatigue life, L_C, is:

$$L_C = 1/0.85[0.4(1/L_1) + 0.4(1/L_2) + 0.2(1/L_3)].$$

SECTION 3 Environmental Conditions

1 General

Spectral-based Fatigue Analysis typically uses environmental data for ocean waves that are given in a "wave scatter" diagram format. The wave data consist of a number of "cells" that represent the probability of occurrence of specific "sea states". Each cell effectively contains three data items:

i) The significant wave height, H_s, (typically in meters)

ii) The characteristic wave period (in seconds)

iii) The probability of occurrence of the sea state

Reference should be made to Appendix 1, which presents the wave scatter diagram data that should be used in the spectral-based fatigue analysis of a vessel classed for "unrestricted service". It can be assumed that there is an equal probability of vessel heading relative to the direction of the waves.

SECTION 4 Motion Analysis and Wave-induced Loads

1 General

This Section gives general criteria on the parameters to be obtained from the vessel motion analysis and the calculation of wave-induced load effects. In the context of a Spectral-based Fatigue Analysis, the main objective of motion and load calculations is the determination of *Response Amplitude Operators* (RAOs), which are mathematical representations of the vessel responses and load effects to unit amplitude sinusoidal waves. The motion and load effects RAOs should be calculated for ranges of wave frequencies and wave headings, as indicated in Subsection 2/3.

Aside from vessel motions, the other wave-induced load effects that should be considered in the Spectral-based Fatigue Analysis are:

• The external wave pressures;

• Internal tank pressures and cargo hold loads due to fluid and cargo accelerations, and

• Inertial forces on the masses of structural components and, as applicable, significant items of equipment.

Additionally, there may be situations where partial models of the structural system are used. In such a case, hull girder shear forces and bending moments should be determined to appropriately represent the boundary conditions at the ends of the partial models.

The general approach used in the calculation methods described below is to calculate total stress response considering both the wave-induced and still-water (static) loads. Subsequently, the still-water stress is deducted from the total, leaving the pure wave-induced stress response. Finally, the fatigue-inducing dynamic stress range is obtained. Alternative methods and formulation that directly produce the dynamic fatigue-inducing stress range may also be used.

Note: Fatigue damage due to the sloshing of fluid in partially filled tanks is not within the scope of the **SFA** classification notation. However, the designer is encouraged to perform and submit such calculations, if deemed important.

3 Initial Balance Check

The motion and load calculations should be performed with respect to static initial conditions representing the vessel geometry, loadings, etc. (see Subsection 2/5). With the input of hull loadings, the hull girder shear force and bending moment distributions in still water should be computed at a sufficient number of transverse sections along the hull's length in order to accurately take into account discontinuities in the weight distribution. A recognized hydrostatic analysis program should be used to perform these calculations. By iteration, the convergence of the displacement, Longitudinal Center of Gravity (LCG) and trim should be checked to meet the following tolerances:

Displacement:	±1%	
Trim:	±0.5 degrees	
Draft:		
	Forward	±1 cm
	Mean	±1 cm
	Aft	±1 cm
LCG:	±0.1% of length	
SWBM:	±5%	

Additionally, the longitudinal locations of the maximum and the minimum still-water bending moments and, if appropriate, that of zero SWBM should be checked to assure proper distribution of the SWBM along the vessel's length.

5   Essential Features of Spectral-based Analysis of Motion and Wave Load

5.1   General Modeling Considerations

There should be sufficient compatibility between the hydrodynamic and structural models so that the "mapping" of fluid pressures onto the structure's finite element model is done appropriately.

For the load component types and structural responses of primary interest, analysis software formulations derived from linear idealizations are deemed to be sufficient. However, the use of enhanced bases for the analysis, especially to incorporate non-linear loads (for example, hull slamming), is encouraged. The adequacy of the employed calculation methods and tools is to be demonstrated to the satisfaction of ABS.

5.3   Diffraction-Radiation Methods

Computations of the wave-induced motions and loads should be carried out using appropriate, proven methods. Preference should be given to the application of seakeeping analysis codes utilizing three-dimensional, potential flow-based diffraction-radiation theory. These codes, based on linear wave and motion assumptions, make use of boundary element methods with constant or higher order sink-source panels over the entire wetted surface of the hull on which the hydrodynamic pressures are computed. All six degrees-of-freedom rigid-body motions of the vessel should be accounted for.

S E C T I O N **5 Wave-induced Load Components**

1 General

Wave-induced loads on a buoyant structure are complicated because, in addition to producing direct forces (e.g., wave pressures on the external surface of the hull), there are indirect force components produced by the rigid body motions of the vessel. The motions result in inertial forces and rotational components of the (quasi-statically considered) loads. These two motion-related load components are referred to below as the "inertial" and "quasi-static" load components.

The treatment of the various load and motion effects is typically done through the use of their real and imaginary parts that are employed separately in structural analyses. In a physical sense, the real and imaginary parts correspond to two wave systems that are 90 degrees out of phase relative to each other.

The following Subsections list the primary wave-induced load components that are to be considered in the Spectral-based Fatigue Analysis of a vessel. Using the methods and calculation tools that are mentioned in Section 4, the Response Amplitude Operators (RAOs) for the listed components should be obtained.

3 External Pressure Component

3.1 Total Hydrodynamic Pressures

The total hydrodynamic pressure should include the direct pressure components due to waves and the components due to hull motions. The components of the hydrodynamic pressure should be determined from the model and calculation procedure mentioned in Section 4.

3.3 Intermittent Wetting *(15 August 2006)*

Ship motion analysis based on linear theory will not predict the non-linear effects near the mean waterline due to intermittent wetting. In actual service, this phenomenon is manifested by a reduction in the number of fatigue cracks at side shell plating stiffeners located near the waterline compared to those about four (4) or five (5) bays below. To take into account the pressure reduction near the mean waterline due to this non-linearity, the following reduction factor can be used:

$$RF = 0.5[1.0 + \tanh(0.35d)]$$

where d is depth, in meters, of the field point below the still-water waterline.

Note: In order to correctly implement the intermittent wetting effects, the size of hydrodynamic panel of side shell near waterline should be appropriately modeled with consideration of longitudinal spacing. It is recommended that the size of panel be no greater than two times of side longitudinal spacing in the vertical direction.

3.5 Pressure Distribution on Finite Element Models

The pressure distribution over a hydrodynamic panel model may be too coarse to be used directly in the structural FEM analysis. Therefore, as needed, the pressure distribution is to be interpolated (3-D linear interpolation) over the finer structural mesh.

5   Internal Load Components

5.1   Tank Pressures

As stated in Subsection 5/1, the vessel motion-related, internal tank pressure is composed of quasi-static and inertial components. The quasi-static component results from the instantaneous roll and pitch of the vessel. The inertial component is due to the acceleration of the fluid caused by vessel motion in six degrees of freedom. The vessel motion should be obtained from analysis performed in accordance with Section 4.

The total internal tank pressure for each of the tank boundary points can be calculated as follows:

$$P = P_o + \rho h_t \{[(g_x + a_x)^2 + (g_y + a_y)^2 + (g_z + a_z)^2]\}^{0.5}$$

where

|		
P	=	total internal tank pressure at a tank boundary point
P_o	=	value of the pressure relief valve setting
ρ	=	density of the fluid cargo or ballast
h_t	=	total pressure head defined by the height of the projected fluid column in the direction to the total acceleration vector
a_x, a_y, a_z	=	longitudinal, lateral and vertical motion wave-induced accelerations relative to the vessel's axis system at a tank boundary point.
g_x, g_y, g_z	=	longitudinal, lateral and vertical instantaneous gravitational accelerations relative to the vessel's axis system at a tank boundary point.

The internal pressure at the tank boundary points can be linearly interpolated and applied to all of the nodes of the structural analysis model defining the tank boundary.

5.3   Dry Bulk Cargo Loads

5.3.1   General

Dry bulk cargo loads in the cargo holds should be determined and applied to the structural analysis model. Both the quasi-static and inertial bulk cargo load components should be included in the analysis. As appropriate, it can be assumed that the hold is full or partially loaded, and there is no relative movement between the hold and the bulk cargo that the hold contains.

As appropriate, the fluid pressure on ballast tank boundaries and the boundaries of a cargo hold carrying water ballast should be considered in the analysis, in accordance with 5/5.1.

FIGURE 1
Hold Boundary Definition

α : Structural Surface Angle
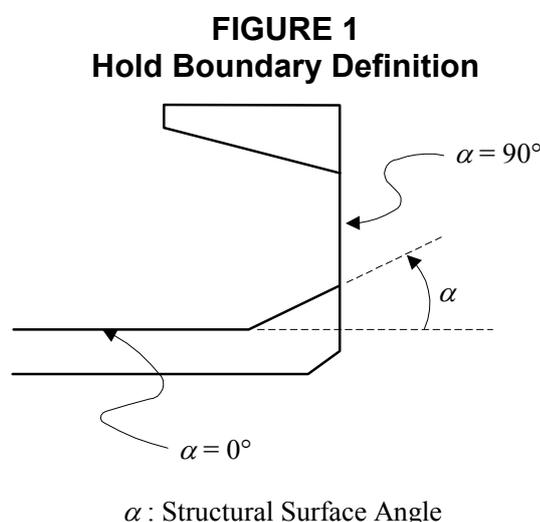
5.3.2   Definitions

α_0 = angle of repose, deg., in accordance with IMO publication, *Code of Safe Practice for Solid Bulk Cargoes*

α = slope of structural surface (0 to 90 deg.)

ρ = the density of the bulk cargo

θ = roll angle, positive starboard down

a_V = local vertical acceleration in ship fixed coordinate system

a_T = local transverse acceleration in ship fixed coordinate system

b = horizontal distance from the centerline of the cargo hold to the point of interest on the hold boundary

g = gravitational acceleration

h = static head to cargo upper surface which may have a shape, vertical distance from the cargo surface to the point of interest on the cargo hold boundary

5.3.3   Pressure Components

The internal bulk load is composed of quasi-static and inertial pressure components. The quasi-static component results from gravity, considering the instantaneous roll and pitch displacements of the vessel. The inertial component is due to the acceleration of the bulk cargo caused by the ship motion in six degrees of freedom. The ship motion should be obtained from the ship motion analysis presented in Section 4.

5.3.4   Quasi-static Components

The bulk load due to gravity can be decomposed into the vertical and horizontal components of the bulk loads.

The vertical load on a unit area of panel is expressed as:
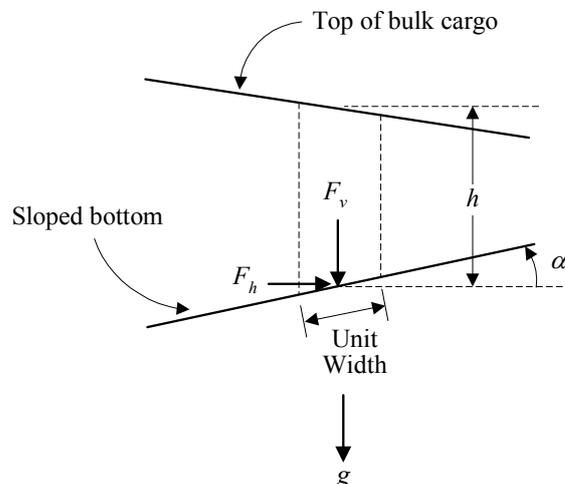
$$F_v = \rho \, gh \cos \alpha$$

where ρ is the density of the bulk cargo.

The horizontal load on a unit area of panel is expressed as:

$$F_h = \rho \, gh \, (1 - \sin \alpha_0) \sin \alpha$$

The above formulas are applicable to a heaped (untrimmed) cargo and also to a (trimmed) flat cargo.

FIGURE 2
Vertical and Horizontal Force Components of Quasi-Static Load

The quasi-static load is further decomposed into the normal and tangential components relative to the boundary surfaces of the cargo hold. The following formulas can be used to calculate the bulk pressures on the bottom, and the sloped and vertical walls of a cargo hold.

The normal load on a unit area of panel is given by:

$$N_s = \rho g h_e [\cos^2 (\alpha - \theta) + (1 - \sin \alpha_0) \sin^2 (\alpha - \theta)]$$

The tangential load on a unit area of panel is given by:

$$T_s = -\rho g h_e [\sin \alpha_0 \sin (\alpha - \theta) \cos (\alpha - \theta)]$$

where

ρ =   density of the bulk cargo

h_e =   effective head of bulk cargo which is the quasi-static pressure head defined by the height of the projected bulk cargo column in the direction of the gravitational acceleration vector at the inclined vessel position. In the case of a flat surface of bulk cargo, it becomes $h/\cos(\theta)$

The inclination of the hold due to roll and pitch of the vessel should be considered in the calculation of the bulk cargo pressure. The direction of gravitational forces in the vessel's fixed coordinate system varies with the roll and pitch, resulting in a change of the pressure head and, correspondingly, the quasi-static pressure.

FIGURE 3
Normal and Tangential Load Components
of Quasi-Static Load in a Rolled Position
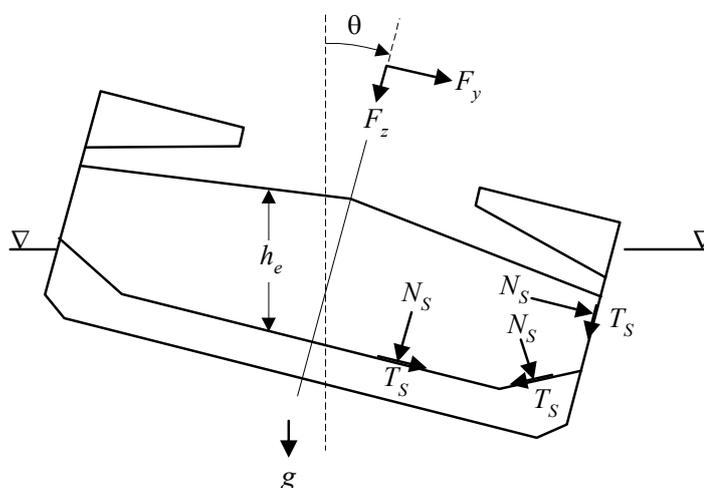
5.3.5   Inertial Components

The inertial components are due to the instantaneous accelerations (longitudinal, transverse and vertical) at the hold boundary points. This total instantaneous internal bulk loading (quasi-static + inertial) for each of the modeled hold boundary points should be obtained.

In this procedure, the vertical, transverse and longitudinal accelerations due to the ship motion are defined in the ship coordinate system. Therefore, transformation of the acceleration to the ship system due to roll and pitch inclinations will not be needed. The bulk cargo loads caused by vertical and transverse accelerations due to the ship motion are described below.

5.3.5(a) Inertial Pressure due to Vertical Accelerations. The pressure due to vertical acceleration is further decomposed into the normal and tangential components relative to the boundary surfaces of the cargo hold.

The normal pressure is given by:

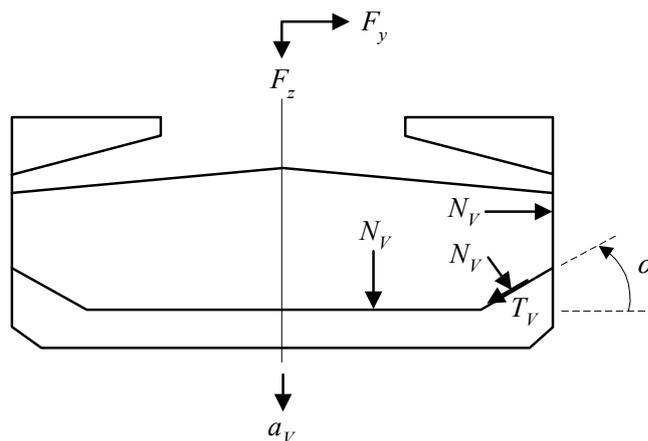$$N_V = \rho\, a_V\, h\, [\cos^2\alpha + (1 - \sin\alpha_0)\sin^2\alpha]$$

The tangential pressure is given by:

$$T_V = -\rho\, a_V\, h\, (\sin\alpha_0 \sin\alpha \cos\alpha)$$

where

N_V = normal component of instantaneous internal bulk cargo pressure at a hold boundary point

T_V = tangential component of instantaneous internal bulk cargo pressure at a hold boundary point

ρ = density of the bulk cargo

FIGURE 4
Pressures Due to Vertical Acceleration

5.3.5(b) Pressure Due to Transverse Acceleration. The pressure due to transverse acceleration is further decomposed into the normal and tangential components relative to the boundary surfaces of the cargo hold.

The normal pressure may be calculated by:

$$N_T = C_n \rho a_T b[\cos^2(90 - \alpha) + (1 - \sin\alpha_0)\sin^2(90 - \alpha)]$$

The tangential pressure may be calculated by:

$$T_T = -C_t \rho a_T b[\sin\alpha_0 \sin(90 - \alpha)\cos(90 - \alpha)]$$

where

N_T = normal component of instantaneous internal bulk cargo pressure at a hold boundary point

T_T = tangential component of instantaneous internal bulk cargo pressure at a hold boundary point

ρ = density of the bulk cargo

$$C_n, C_t =$$ friction reduction factors due to transverse acceleration for normal component and tangential component, respectively. These factors, in general, depend on the type of bulk cargo and should be determined based on reliable test data. If no such data are available, the following values may be used:

$$= \quad 0.35 \quad \text{for ore}$$

$$= \quad 0.6 \quad \text{for grain}$$

FIGURE 5
Pressures Due to Transverse Acceleration
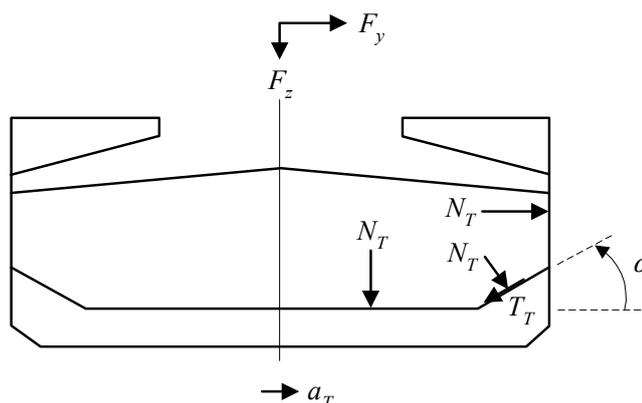
The total pressure can be obtained by adding the quasi-static component and the inertial bulk cargo pressure.

The formulas in 5/5.3.4 and 5/5.3.5 can be used for the pitched condition with longitudinal acceleration by replacing the roll angle, transverse acceleration and transverse pressure with pitch angle, longitudinal acceleration and longitudinal pressure, respectively.

5.5   Container Loads

5.5.1   General

The loads on hull structure that result from containers in a cargo hold, where all containers in the hold are restrained by cell guides, and on deck should be determined and applied to the structural analysis model. Quasi-static and inertial container load components should be included in the analysis. It is assumed that there is no relative movement between the hull and containers.

As applicable, ballast water and fuel oil pressure loadings (primarily on double bottom and wing tank boundary structure) should be considered in the analysis, in accordance with 5/5.1.

5.5.2   Load Components

The container load is composed of quasi-static and inertial load components. The quasi-static component results from gravity, considering the instantaneous roll and pitch displacements of the vessel. The inertial component is due to the acceleration of the container cargo caused by the ship motion in six degrees of freedom. The ship motion should be obtained from the ship motion analysis presented in Section 4.

5.5.3   Quasi-Static Load

The quasi-static load due to gravity can be decomposed into the vertical and transverse components of the container loads.

The vertical load on the inner bottom and on the deck due to containers is expressed as:

$$F_V = w \cos \theta$$

where

w = weight of a container

θ = roll angle

The vertical load due to a stack of containers may be summed and applied to the appropriate nodes on the inner bottom plating. The total vertical load due to the containers on deck may be applied to the appropriate nodes on the hatch coaming top plates.

The transverse load is expressed as:

$F_T = w \sin \theta$

The transverse load due to containers may be distributed to the appropriate nodes on the bulkhead structure via container cell guides. The total transverse load due to the containers on deck may be applied to the suitable nodes on the hatch coaming top plates, considering, as appropriate, the effects of the container lashing system.

The formulas given above can also be used for the pitched condition by replacing the roll angle with the pitch angle.

The inclination of the hold due to the roll and pitch of the vessel should be considered in the calculation of the container cargo load. The direction of gravitational forces in the vessel's fixed coordinate system varies with the roll and pitch, resulting in a change of the magnitude of vertical, transverse and longitudinal loads.

FIGURE 6
Vertical and Transverse Force Components of Static Load
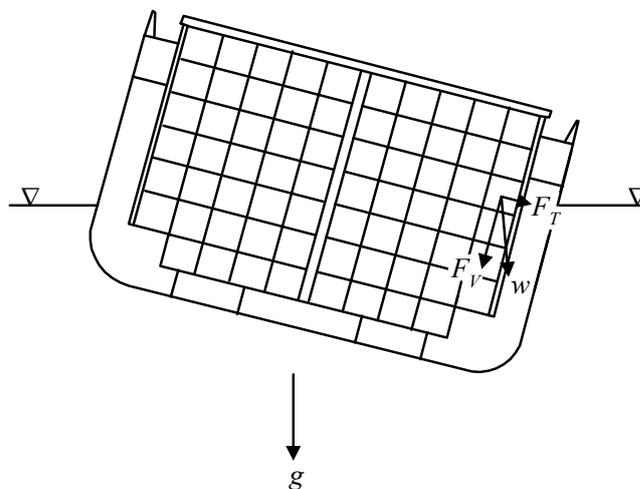
5.5.4 Inertial Loads

The inertial load is due to the instantaneous accelerations (longitudinal, transverse and vertical) of the container as calculated at the center of gravity (CG) of a container. This total instantaneous container loading (quasi-static + inertial) should be calculated and applied to the appropriate nodes of the structural analysis model.

In this procedure, the vertical, transverse and longitudinal accelerations due to the ship motion should be defined in the ship coordinate system. Therefore, transformation of the acceleration to the ship system due to roll and pitch inclinations will not be needed. The container loads caused by vertical and transverse accelerations due to the ship motion are described below.

The inertial load due to a vertical acceleration is calculated by:

$$N_V = w\, a_v/g$$

where

N_V = vertical component of instantaneous container load

a_v = vertical acceleration

g = gravitational acceleration

The inertial load due to transverse acceleration is calculated by:

$$N_T = w\, a_T/g$$

where

N_T = normal component of instantaneous container load at a hold boundary point

a_T = transverse acceleration

The approach outlined above can also be used for a pitched condition with longitudinal acceleration by replacing the roll angle and transverse acceleration with pitch angle and longitudinal acceleration, respectively.

The vertical and transverse components of the motion-induced load should be applied to the structural analysis model, as described in 5/5.5.3. The total load is obtained by summing the quasi-static component and the inertial container cargo load.

FIGURE 7
Inertial Loads Due to Acceleration
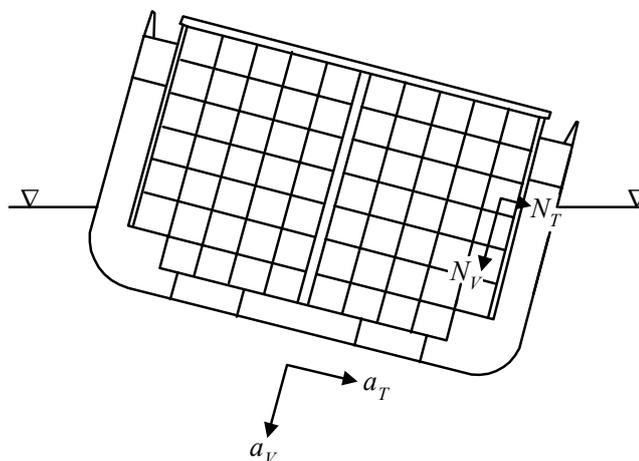
7 Loads from the Motions of Discrete Masses

Vessel motions produce loads acting on the masses of "lightship" structure and equipment. There are quasi-static and inertial components which can be obtained in the following manner. The motion-induced acceleration, A_t, is determined for each discrete mass from the formula:

$$A_t = (R \times \Phi)\omega^2 + a$$

where

R = distance vector from the hull's CG to the point of interest

Φ = rotational motion vector

$\times$ = cross product between the vectors

a = translation acceleration vector

ω = relevant frequency

Using the real and imaginary parts of the complex accelerations calculated above, the motion-induced load is computed by:

$F = m (A_t)$

where m is the discrete mass under consideration.

The real and imaginary parts of the motion-induced loads from each discrete mass in all three directions are calculated and applied to the structural model.

SECTION 6   **Loading for Global Finite Element Method (FEM) Structural Analysis Model**

1   General

For each heading angle and wave frequency at which the structural analysis is performed (see Subsection 2/3), two load cases corresponding to the real and imaginary parts of the frequency regime wave-induced load components are to be analyzed. Then, for each heading angle and wave frequency, the stress range transfer function, $H_\sigma(\omega|\theta)$, is obtained for each considered Base Vessel Loading Condition.

3   Number of Load Cases

The number of combined load cases for each Base Vessel Loading Condition can be relatively large. When the structural analysis is performed for 33 frequencies (0.2 to 1.80 rad/s at 0.05 increment) and 12 wave headings (0 to 360 degree at 30 degree increment), the number of combined Load Cases is 792 (considering separate real and imaginary cases). If there are two (2) Base Vessel Loading Conditions, the total number of load cases is (2 · 792) = 1584.

5   Equilibrium Check

The applied hydrodynamic external pressure should be in equilibrium with the other loads applied. The unbalanced forces in three global directions for each load case should be calculated and checked. For head sea condition, the unbalanced force should not exceed one percent of the displacement. For oblique and beam sea condition, it should not exceed two percent of the displacement. These residual forces could be balanced by adding suitably distributed inertial forces [so called "inertial relief"] before carrying out the FEM structural analysis.

SECTION 7 Structural Modeling and Analysis

1 General

The stress range transfer function, $H_\sigma(\omega|\theta)$, for a location where the fatigue strength is to be evaluated should be determined by the finite element method (FEM) of structural analysis, using a three dimensional (3-D) model representing the entire hull structure. This analysis may produce results of sufficient accuracy, but more typically, it is also necessary to perform fine mesh analyses of local areas, using boundary a condition determined from the whole ship analysis. The load cases to be used in the analysis should be those obtained in accordance with Section 6.

As necessary to evaluate the fatigue strength of local structure, finer mesh FEM analyses are also to be performed. Results of nodal displacements or forces obtained from the overall 3-D analysis model are to be used as boundary conditions in the subsequent finer mesh analysis of local structures.

Specialized fine mesh FEM analysis is required in the determination of stress concentration factors associated with the "hot-spot" fatigue strength evaluation procedures (see Subsection 7/7).

Note: Reference should be made to additional ABS Guidance on the expected modeling and analysis of vessel structure, e.g., the ABS *Finite Element Analysis Guidance* that is provided with the SafeHull software. While there are significant differences in the extent of structural model described here and the partial hull model pursued in SafeHull, numerous detailed modeling considerations are shared, such as element types, mesh sizes, dependence between local and global models, etc.

3 3-D Global Analysis Modeling

The global structural and load modeling should be as detailed and complete as practicable. For the Spectral-based Fatigue Analysis of a new-build structure, gross scantlings are ordinarily used.

In making the model, a judicious selection of nodes, elements and degrees of freedom is to be made to represent the stiffness and inertial properties of the hull. Lumping of plating stiffeners, use of equivalent plate thickness and other techniques may be used to keep the size of the model and required data generation within manageable limits.

The finite elements, whose geometry, configuration and stiffness closely approximate the actual structure, are of three types:

i)   Truss or bar elements with axial stiffness only

ii)   Beam elements with axial, shear and bending stiffness

iii)   Membrane and bending plate elements, either triangular or quadrilateral

5 Analyses of Local Structure

More refined local stress distributions should be determined from the fine mesh FEM analysis of local structure. In the fine mesh models, care is to be taken to accurately represent the structure's stiffness as well as its geometry. Boundary displacements obtained from the 3-D global analysis are to be used as boundary conditions in the fine mesh analysis. In addition to the boundary constraints, the pertinent local loads should be reapplied to the fine mesh models.

7   Hot Spot Stress Concentration

When employing the so-called "Hot-Spot" Stress Approach (for example, to determine the fatigue strength at the toe of a fillet weld), it is necessary to establish a procedure to be followed to characterize the expected fatigue strength. The two major parts of the procedure are (a) the selection of an S-N Data Class (see Section 8) that applies in each situation; and (b) specifying the fine mesh FEM model adjacent to the weld toe detail and how the calculated stress distribution is extrapolated to the weld toe (hot-spot) location. Section 7, Figure 1 shows an acceptable method that can be used to extract and interpret the "near weld toe" element stresses and to obtain a (linearly) extrapolated stress at the weld toe. Element sizes in such cases near the weld toe would be close to the thickness of the plating. When stresses are obtained in this manner, the use of the E class S-N data (see Appendix 2) is considered to be most appropriate.

FIGURE 1
Definition of Hot Spot Stress
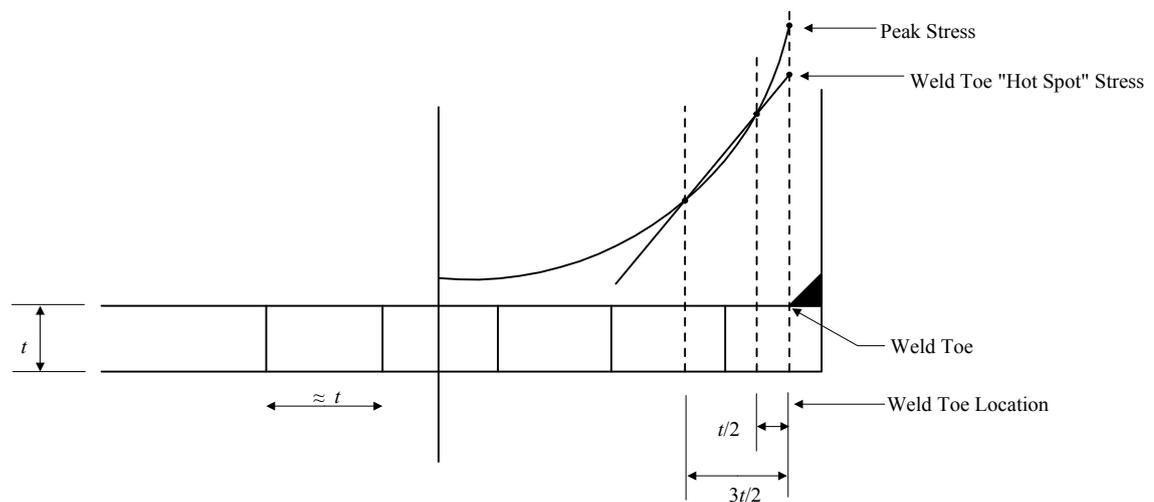
Weld hot spot stress can be determined from linear extrapolation to the weld toe, using calculated stresses at $t/2$ and $3t/2$ from the weld toe. Defining stresses are the principal surface stresses (considering a "bending plate" element type) at the locations shown. A description of the numerical extrapolation procedure is given in Appendix 5C-1-A1 of the *Steel Vessel Rules* (and also in other, similar locations of the *Steel Vessel Rules.)*

SECTION 8 Fatigue Strength

1 General

The previous sections of this Guide have addressed establishing the stress range (demand) for locations in the structure for which the adequacy of fatigue strength is to be evaluated. The capacity of a location to resist fatigue damage is characterized by the use of S-N Data, which are described below. Refer to the Appendix 2 of this Guide and Part 5C of the *Steel Vessel Rules* concerning the S-N Data recommended by ABS.

Using the S-N approach, fatigue strength (capacity) is usually characterized in one of two ways. One way is called a *nominal stress approach*. In this approach, the acting variable stress range (demand) is considered to be obtained adequately from the nominal stress distribution (which may include so-called "geometric" stress concentration effects) in the area surrounding the particular location for which the fatigue life is being evaluated. The other way of characterizing fatigue strength (capacity) at a location is the *"hot-spot" approach* (see Subsection 7/7). The hot-spot approach is needed for locations where complicated geometry or relatively steep local stress gradients would make the use of the nominal stress approach inappropriate or questionable.

Reference should be made to Part 5C of the *Steel Vessel Rules* for further explanation and application of these two approaches and for guidance on the categorization of structural details into the various S-N data classes.

3 S-N Data *(15 November 2009)*

To provide a ready reference, the S-N Data recommended by ABS are given in Appendix 2 of this Guide. (***Note:*** source United Kingdom's Dept of Energy (HSE) Guidance Notes, 4th Edition.)

There are various adjustments (reductions in capacity) that may be required to account for factors such as a lack of corrosion protection (coating) of structural steel and relatively large plate thickness. The imposition of these adjustments on fatigue capacity will be in accordance with ABS practice for vessels.

There are other adjustments that could be considered to increase fatigue capacity above that portrayed by the cited S-N data. These include adjustments for compressive "mean stress" effects, a high compressive portion of the acting variable stress range and the use of "weld improvement" techniques. The use of a weld improvement technique, such as weld toe grinding or peening to relieve ambient residual stress, can be effective in increasing fatigue life. However, credit should not be taken of such a weld improvement in the design phase of the structure. Consideration for granting credit for the use of weld improvement techniques should be reserved for situations arising during construction, operation or future reconditioning of the structure. An exception may be made if the target design fatigue life cannot be satisfied by other preferred design measures such as refining layout, geometry, scantlings and welding profile to minimize fatigue damage due to high stress concentrations. Grinding or ultrasonic peening can be used to improve fatigue life in such cases. The calculated fatigue life is to be greater than 15 years excluding the effects of life improvement techniques. Where an improvement technique is applied, full details of the grinding standard including the extent, profile smoothness particulars, final weld profile, and improvement technique workmanship and quality acceptance criteria are to be clearly shown on the applicable drawings and submitted for review together with supporting calculations indicating the proposed factor on the calculated fatigue life.

Grinding is preferably to be carried out by rotary burr and to extend below the plate surface in order to remove toe defects and the ground area is to have effective corrosion protection. The treatment is to produce a smooth concave profile at the weld toe with the depth of the depression penetrating into the plate surface to at least 0.5 mm below the bottom of any visible undercut. The depth of groove produced is to be kept to a minimum, and, in general, kept to a maximum of 1 mm. In no circumstances is the grinding depth to exceed 2 mm or 7% of the plate gross thickness, whichever is smaller. Grinding has to extend to areas well outside the highest stress region.

The finished shape of a weld surface treated by ultrasonic peening is to be smooth and all traces of the weld toe are to be removed. Peening depth below the original surface is to be maintained at least 0.2 mm. Maximum depth is generally not to exceed 0.5 mm.

Provided these recommendations are followed, an improvement in fatigue life by grinding or ultrasonic peening up to a maximum of 2 times may be granted.

SECTION 9 Fatigue Life (Damage) Calculation and Acceptance Criteria

1 General

Mathematically, Spectral-based Fatigue Analysis begins after the determination of the stress transfer function. Wave data are then incorporated to produce stress-range response spectra, which are used to describe probabilistically the magnitude and frequency of occurrence of local stress ranges at the locations for which fatigue strength is to be calculated. Wave data are represented in terms of a wave scatter diagram and a wave energy spectrum. The wave scatter diagram consists of sea-states, which are short-term descriptions of the sea in terms of joint probability of occurrence of a significant wave height, H_s, and a characteristic period.

An appropriate method is to be employed to establish the fatigue damage resulting from each considered sea state. The damage resulting from individual sea states is referred to as "short-term". The total fatigue damage resulting from combining the damage from each of the short-term conditions can be accomplished by the use of a weighted linear summation technique (i.e., Miner's Rule).

Appendix 3 contains a detailed description of the steps involved in a suggested Spectral-based Fatigue Analysis method that follows the basic elements mentioned above. ABS should be provided with background and verification information that demonstrates the suitability of the analytical method employed.

3 Acceptance Criteria

The required fatigue strength can be specified in several ways, primarily depending on the evaluation method employed. For the Spectral-based approach, it is customary to state the minimum required fatigue strength in terms of a Damage ratio (D) or minimum target Life (L). The latter is employed in this Guide and in the *Steel Vessel Rules*.

The Spectral-based Fatigue Analysis of a vessel that is classed for "unrestricted service" should be based on the "wave scatter" diagram data given below.

TABLE 1
ABS Wave Scatter Diagram for Unrestricted Service Classification *(1 December 2007)*

* Wave heights taken as significant values, H_s

** Wave periods taken as zero crossing values, T_Z

Wave Height (m)*	Wave Period (sec)**											Sum Over All Periods
	3.50	4.50	5.50	6.50	7.50	8.50	9.50	10.50	11.50	12.50	13.50	
0.5	8	260	1344	2149	1349	413	76	10	1			5610
1.5		55	1223	5349	7569	4788	1698	397	69	9	1	21158
2.5		9	406	3245	7844	7977	4305	1458	351	65	10	25670
3.5		2	113	1332	4599	6488	4716	2092	642	149	28	20161
4.5			30	469	2101	3779	3439	1876	696	192	43	12625
5.5			8	156	858	1867	2030	1307	564	180	46	7016
6.5			2	52	336	856	1077	795	390	140	40	3688
7.5			1	18	132	383	545	452	247	98	30	1906
8.5				6	53	172	272	250	150	65	22	990
9.5				2	22	78	136	137	90	42	15	522
10.5				1	9	37	70	76	53	26	10	282
11.5					4	18	36	42	32	17	7	156
12.5					2	9	19	24	19	11	4	88
13.5					1	4	10	14	12	7	3	51
>14.5					1	5	13	19	19	13	7	77
Sum over All Heights	8	326	3127	12779	24880	26874	18442	8949	3335	1014	266	100000

APPENDIX 2   Basic Design S-N Curves

FIGURE 1
S-N Curves
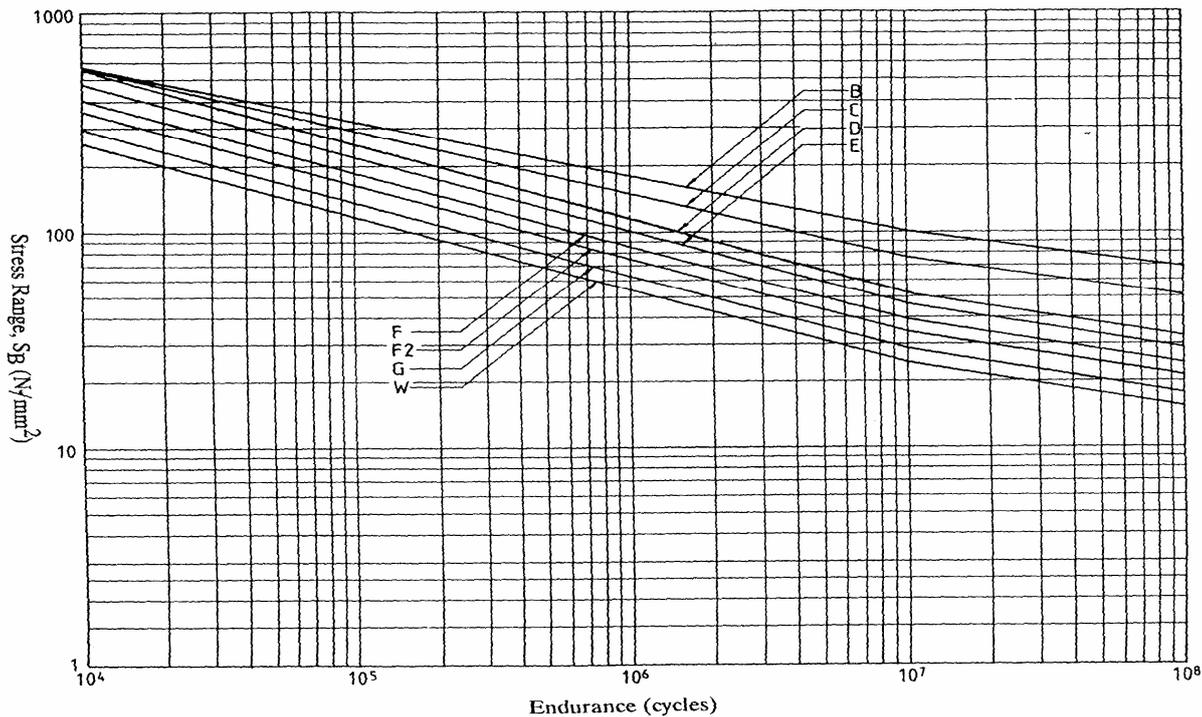
The S-N Curves are represented by the following equation:

$$S^m N = A$$

where

S =   stress range

N =   number of cycles to failure

A, m =   parameters representing the intercept and inverse slope of the upper (left) portion of the S-N Curve. These change at $N = 10^7$ cycles to C and r, respectively. Values of these parameters are given in the following table.

TABLE 1
Parameters For Basic S-N Design Curves *(1 December 2007)*

	$N \leq 10^7$		$N > 10^7$	
Class	*A* (For MPa units)	*m*	*C* (For MPa units)	*r*
B	1.013×10^{15}	4	1.020×10^{19}	6
C	4.227×10^{13}	3.5	2.584×10^{17}	5.5
D	1.519×10^{12}	3	4.239×10^{15}	5
E	1.035×10^{12}	3	2.300×10^{15}	5
F	6.315×10^{11}	3	9.975×10^{14}	5
F2	4.307×10^{11}	3	5.278×10^{14}	5
G	2.477×10^{11}	3	2.138×10^{14}	5
W	1.574×10^{11}	3	1.016×10^{14}	5

Refer to Part 5C of the *Steel Vessel Rules* on the categorization of structural details into the indicated classes.

Notes for Application of Classes:

Class B: Parent material with automatic flame-cut edges ground to remove flame cutting drag line.

Class C: Parent material with automatic flame-cut edges and full penetration butt welds ground flush in way of hatch corners in container carriers or similar deck areas in other vessel types.

Class D: Full penetration butt welds in way of hatch corners in container carriers or similar deck areas in other vessel types.

APPENDIX **3   Outline of a Closed Form Spectral-based Fatigue Analysis Procedure**

Notes:

(1)   This Appendix is referred to in Section 9. It is provided to describe the formulations comprising a Spectral-based Fatigue Analysis approach, which can be employed to satisfy the criteria to obtain the **SFA (*years*)** Classification notation.   However, it is often at this formulation level that valid variations of a method may be introduced. For that reason, it is emphasized that the contents of this Appendix are provided primarily to illustrate principle rather than to give mandatory steps for the Spectral-based Fatigue method.

(2)   The procedure described below considers the use of a wave scatter diagram representative of a one-year period (i.e., as in Appendix 1). Where a different base period for the wave scatter diagram is employed, the procedure must be suitably modified.

1   General

In the "short-term closed form" approach described below, the stress range is normally expressed in terms of probability density functions for different short-term intervals corresponding to the individual cells or bins of the wave scatter diagram. These short-term probability density functions are derived by a spectral approach based on the Rayleigh distribution method, whereby, it is assumed that the variation of stress is a narrow-banded random Gaussian process. To take into account effects of swell, which are not accounted for when the wave environment is represented by the scatter diagram, Wirsching's "*rainflow correction*" factor is applied in the calculation of short-term fatigue damage. Having calculated the short-term damage, the total fatigue damage is calculated through their weighted linear summation (using Miner's rule). Mathematical representations of the steps of the Spectral-based Fatigue Analysis approach just described are given next.

3   Key Steps in Closed Form Damage Calculation

1.   Determine the complex stress transfer function, $H_\sigma(\omega|\theta)$, at a structural location of interest for a particular load condition. This is done in a direct manner where structural analyses are performed for the specified ranges of wave frequencies and headings, and the resulting stresses are used to explicitly generate the stress transfer function.

2.   Generate a stress energy spectrum, $S_\sigma(\omega|H_s, T_z, \theta)$, by scaling the wave energy spectrum $S_\eta(\omega|H_s, T_z)$ in the following manner:

$$S_\sigma(\omega|H_s, T_z, \theta) = |H_\sigma(\omega|\theta)|^2 \, S_\eta(\omega|H_s, T_z) \dotfill (1)$$

3.   Calculate the spectral moments. The nth spectral moment, m_n, is calculated as follows:

$$m_n = \int_0^\infty \omega^n \, S_\sigma(\omega|H_s, T_z, \theta) \, d\omega \dotfill (2)$$

Most fatigue damage is associated with low or moderate seas, hence, confused short-crested sea conditions must be allowed. Confused short-crested seas result in a kinetic energy spread, which is modeled using the cosine-squared approach, $(2/\pi) \cos^2\theta$. Generally, cosine-squared spreading is assumed from +90 to –90 degrees on either side of the selected wave heading. Applying the wave spreading function modifies the spectral moment as follows:

$$m_n = \int_0^\infty \sum_{\theta'=\theta-90}^{\theta'=\theta+90} \left(\frac{2}{\pi}\right) \cos^2\theta' \cdot \omega^n S_\sigma(\omega \,|\, H_s, T_z, \theta) d\omega \quad\text{...} (3)$$

4.   *(1 December 2007)* Using the spectral moments, the Rayleigh probability density function (pdf) describing the short term stress-range distribution, the zero up-crossing frequency of the stress response and the bandwidth parameter used in calculating Wirsching's "rainflow correction" are calculated as follows:

Rayleigh pdf:

$$g(s) = \frac{s}{4\sigma^2} \exp\left[-\left(\frac{s}{2\sqrt{2}\sigma}\right)^2\right] \quad\text{...} (4)$$

Zero-up crossing frequency, in Hz:

$$f = \frac{1}{2\pi} \sqrt{\frac{m_2}{m_0}} \quad\text{...} (5)$$

Bandwidth Parameter:

$$\varepsilon = \sqrt{1 - \frac{m_2^2}{m_0 m_4}} \quad\text{...} (6)$$

where

      s =   stress range (twice the stress amplitude)

      σ = $\sqrt{m_0}$

   m_0, m_2, m_4 =   spectral moments

5.   *(1 December 2007)* Calculate cumulative fatigue damage based on Palmgren-Miner's rule, which assumes that the cumulative fatigue damage (D) inflicted by a group of variable amplitude stress cycles is the sum of the damage inflicted by each stress range (d_i), independent of the sequence in which the stress cycles occur:

$$D = \sum_{i=1}^J d_i = \sum_{i=1}^J \frac{n_i}{N_i} \quad\text{...} (7)$$

where

      n_i =   number of stress cycles of a particular stress range

      N_i =   average number of loading cycles to failure under constant amplitude loading at that stress range according to the relevant S-N curve

      J =   number of considered stress range intervals

Failure is predicted to occur when the cumulative damage (D) over J exceeds a critical value equal to unity.

The short term damage incurred in the i-th sea-state, assuming a S-N curve of the form $N = AS^{-m}$, is given by:

$$D_i = \left(\frac{T}{A}\right) \int_0^\infty (k_t k_{ms} k_h s)^m f_{0i} p_i g_i ds \quad ....(8)$$

where

D_i	=	damage incurred in the i-th sea-state
k_h	=	a factor for high tensile steel, applicable to parent material only
	=	1.000 for mild steel or welded connections
	=	0.926 for H32 steel
	=	0.885 for H36 steel
	=	0.870 for H40 steel
k_t	=	a factor for thickness effect, which is not applicable to the longitudinal stiffeners which are of flat bars or bulb plates
	=	$\left(\dfrac{t}{22}\right)^n$ for $t \geq 22$ mm
	=	1.0 for $t < 22$ mm
n	=	0.25 for cruciform joints, transverse T joints, plates with transverse attachments
	=	0.20 for transverse butt welds
	=	0.10 for butt welds ground flush, base metal, longitudinal welds or attachments

If it can be conclusively established that the detail under consideration is always subject to a mean stress of σ_{ms}, D is to be adjusted by a factor k_{ms}.

k_{ms}	=	a factor for mean stress effect, which is
	=	1.0 for $\sigma_{ms} > s_4/2$
	=	$0.85 + 0.3 \sigma_{ms}/s_4$ for $-s_4/2 \leq \sigma_{ms} \leq s_4/2$
	=	0.7 for $\sigma_{ms} < -s_4/2$
σ_{ms}	=	mean stress
s_4	=	long-term stress range corresponding to the representative probability level of 10^{-4}
m, A	=	physical parameters describing the S-N curve
T	=	design life, in seconds
f_{0i}	=	zero-up-crossing frequency of the stress response, Hz
p_i	=	joint probability of H_s and T_z
g_i	=	probability density function governing s in the i-th sea state
s	=	specific value of stress range

Summing D_i over all of the sea-states in the wave scatter diagram leads to the total cumulative damage, D. Therefore:

$$D = (k_h k_t k_{ms})^m \left(\frac{f_0 T}{A} \right) \int_0^\infty s^m \left[\sum_{i-1}^M f_{0i} p_i g_i / f_0 \right] ds \quad \text{...} \quad (9)$$

where

$\qquad D \qquad = \qquad$ total cumulative damage

$\qquad f_0 \qquad = \qquad$ "average" frequency of s over the lifetime

$\qquad \qquad \qquad = \qquad \Sigma_i\, p_i f_{0i}$ (where the summation is done from $i = 1$ to M, the number of considered sea-states)

Introducing long-term probability density function, $g(s)$ of the stress range as:

$$g(s) = \frac{\displaystyle\sum_i f_{0i} p_i g_i}{\displaystyle\sum_i f_{0i} p_i} \quad \text{..} \quad (10)$$

and

$\qquad N_T =$ total number of cycles in design life $= f_0 T$

the expression for total cumulative damage, D, can be rewritten as:

$$D = (k_h k_t k_{ms})^m \frac{N_T}{A} \int_0^\infty s^m g(s) ds \quad \text{...} \quad (11)$$

6.   If the total number of cycles, N_T, corresponds to the required minimum Design Life of 20 years, the Calculated Fatigue Life would then be equal to 20/D. Increasing the design life to higher values can be done accordingly. The fatigue safety check is to be done in accordance with the applicable Rules where factors of safety (or Fatigue Design Factors) are specified.

5   Closed Form Damage Expression *(1 December 2007)*

For all one-segment linear S-N curves, the closed form expression of damage, D, as given by equation 9, is as follows:

$$D = \frac{T}{A} (2\sqrt{2})^m \ \Gamma(m/2 + 1) \sum_{i=1}^M \lambda(m, \varepsilon_i) f_{0i} p_i (k_h k_t k_{ms} \sigma_i)^m \quad \text{...} \quad (12)$$

where

$\qquad \Gamma \qquad = \qquad$ complete gamma function with the argument $(m/2 + 1)$

$\qquad \sigma_i \qquad = \qquad \sqrt{m_0}$ for the i-th considered sea state

$\qquad \lambda \qquad = \qquad$ rainflow factor of Wirsching and is defined as:

$\qquad \qquad \qquad \lambda(m, \varepsilon_i) = a(m) + [1 - a(m)][1 - \varepsilon_i]^{b(m)} \text{...}$ (13)

where

$\qquad a(m) \quad = \qquad 0.926 - 0.033m$

$\qquad b(m) \quad = \qquad 1.587m - 2.323$

$\qquad \varepsilon_i \qquad = \qquad$ Spectral Bandwidth (equation 6)

For bi-linear S-N curves (see Appendix 2) where the negative slope changes at point $Q = (N_Q, S_Q)$ from m to $r = m + \Delta m$ ($\Delta m > 0$) and the constant A changes to C, the expression for damage, as given in equation 12, is as follows:

$$D = \frac{T}{A}\,(2\sqrt{2})^m\,\Gamma(m/2 + 1)\,\sum_{i=1}^{M}\lambda(m,\varepsilon_i)\mu_i f_{0i}\, p_i\,(k_h k_t k_{ms}\sigma_i)^m \quad\text{...(14)}$$

where μ_i is the endurance factor having its value between 0 and 1 and measuring the contribution of the lower branch to the damage. It is defined as:

$$\mu_i = 1 - \frac{\displaystyle\int_0^{S_Q} s^m g_i\, ds - \left(\frac{A}{C}\right)\!\left[(k_h k_t k_{ms})^{\Delta m}\right]\!\int_0^{S_Q} s^{m+\Delta m} g_i\, ds}{\displaystyle\int_0^{\infty} s^m g_i\, ds} \quad\text{...(15)}$$

If $g(s)$ is a Rayleigh distribution, then μ_i is:

$$\mu_i = 1 - \frac{\Gamma_0(m/2+1, v_i) - (1/v_i)^{\Delta m/2}\left[(k_h k_t k_{ms})^{\Delta m}\right]\Gamma_0(r/2+1, v_i)}{\Gamma(m/2+1)} \quad\text{.................................(16)}$$

where

$$v_i \quad = \quad \left(\frac{S_Q}{2\sqrt{2}\sigma_i}\right)^2$$

$\Gamma_0 \quad = \quad$ incomplete gamma function and is

$$\Gamma_0(a,x) = \int_0^x u^{a-1}\exp(-u)\, du$$

See steps 2-6 above, regarding the fatigue safety check.