GUIDANCE NOTES ON

DYNAMIC ANALYSIS PROCEDURE FOR SELF-ELEVATING DRILLING UNITS

JANUARY 2004

American Bureau of Shipping
Incorporated by Act of Legislature of the State of New York 1862

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Foreword

The guidance contained herein should be used in conjunction with the ABS *Rules for Building and Classing Mobile Offshore Drilling Units* for the purpose of ABS Classification of a Self-Elevating Drilling Unit. The guidance indicates acceptable practice in a typical case for types of designs that have been used successfully over many years of service. The guidance may need to be modified to meet the needs of a particular case, especially when a novel design or application is being assessed. The guidance should not be considered mandatory, and in no case is this guidance to be considered a substitute for the professional judgment of the designer or analyst. In case of any doubt about the application of this guidance ABS should be consulted.

The *self-elevating drilling unit* is referred to herein as "SEDU", the ABS *Rules for Building and Classing Mobile Offshore Drilling Units*, are referred to as the "*MODU Rules*".

ABS welcomes comments about this guidance, especially suggestions for improvement. These can be sent electronically to *www.rdd@eagle.org*.

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GUIDANCE NOTES ON THE

DYNAMIC ANALYSIS PROCEDURE FOR SELF-ELEVATING DRILLING UNITS

CONTENTS

SECTION 1 Introduction

1 Background

These Guidance Notes present acceptable practice for an important aspect in the Classification of self-elevating drilling units (SEDUs). The technical criteria contained in these Guidance Notes are based on the results of a Joint Industry Project sponsored by Owners, Designers, Builders, Operators and Classification Societies. The criteria were subsequently published as Reference 1. That reference is specifically aimed at providing assessment criteria for the site-specific use of the SEDU.

The fundamental difference between site-specific evaluation and Classification is that the latter is not site-specific in nature. Instead, the Owner specifies conditions for which the unit is to be reviewed for Classification. The basic dimensions of the envelope of conditions that the Owner may specify for Classification are:

i) Water depth (plus air gap and penetration depth into the seabed)

ii) Environmental conditions of wind, wave and current

iii) Total elevated load

iv) Spudcan-soil rotational stiffness

(The last item being a consideration introduced by ABS in 2003 when dynamic response is being assessed for Classification.)

Therefore, a major theme of these Guidance Notes is to clarify the portions of the criteria in Reference 1 that should be applied without modification and the portions of the criteria that may need to be adapted for Classification purposes.

3 Basic Concepts of the Inclusion of Dynamic Effects into Structural Analysis

Because the natural period of an SEDU is typically in the range of 5 to 15 seconds, there may be a concern that there will be dynamic amplification (resonance) with waves in this period range. It is therefore often desirable to account for the dynamic effects of the SEDU in the elevated condition due to waves (and waves with current).

The basic approach most commonly used to include dynamic effects into structural analysis is characterized as a "quasi-static" method, which entails a two-step procedure. In the first step, a *Dynamic Analysis* model of the structural system is analyzed. Then, the static response to the same loads is obtained using the same model. A Dynamic Amplification Factor (DAF) is obtained as the ratio of the extreme of a response when dynamics is considered to the extreme of the same response statically considered. DAFs can be quantified for various structural responses, such as the global overturning moment of the unit, base shear force or the lateral displacement of the elevated hull (i.e., surge and sway). From the DAFs, an "inertial load set" is established that simulates the dynamic effects. The loads considered to produce the dynamic response are those induced by waves or waves acting with current. Usually, it is sufficient that the level of structural system idealization used to determine DAFs is, as often described, an "equivalent model", which is an "equivalent 3-leg idealization" coupled with an "equivalent hull structural model". The need to appropriately account for the stiffness of the leg-to-hull interaction and foundation/soil interaction adds some minor complexity to this simplified modeling approach.

In the second step, the "inertial load set" is imposed, along with all of the other coexisting loads, onto the usual, detailed static structural model that is used to perform the "unity checking" for structural acceptance based on the Rules. Because this model now includes the "inertial load set" to simulate the dynamic response, it is often also referred to as the *Quasi-Static* model.

The two-step procedure is summarized as:

i) Use an "equivalent" model to perform a random wave dynamic analysis deriving the inertial load set caused by wave-induced structural dynamics and

ii) Use a "detailed" model to perform, with static gravity and wind loads and quasi-static wave loads plus the derived inertial load set, a static structural analysis deriving the stresses for unity checks in accordance with the ABS strength requirements in the *MODU Rules* for the leg chords, braces and the jacking pinions.

The flowchart of the two-step procedure is shown in Section 1, Figure 1. More details on the modeling procedure and the determination and application of the inertial load set are given in these Guidance Notes as follows:

Specification of Wave Parameters and Spudcan/Soil Stiffness	Section 2
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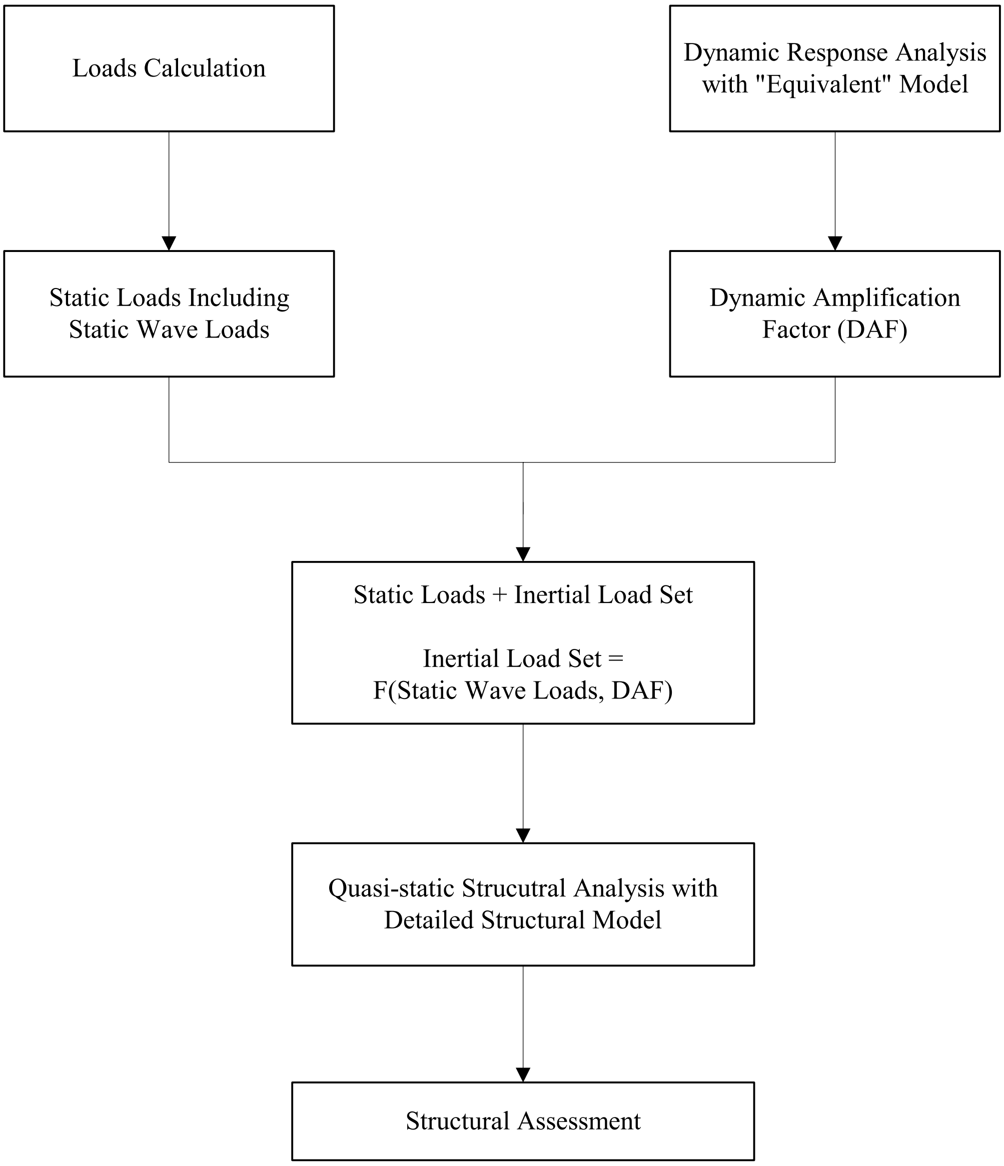
Dynamic Analysis Methods	Section 4
Application of the Inertial Load set	Section 5

5 Exception

If the estimated DAF of the unit is smaller than 1.05, the dynamic magnification may be neglected. The method that may be applied to estimate DAF for this purpose is the SDOF approach indicated in 4/5.1.

FIGURE 1
The Flowchart of the Two-Step Procedure



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SECTION 2 Specification of Wave Parameters and Spudcan/Soil Stiffness

1 Introduction

Environmental and geotechnical data are inherent to site-specific design and analysis. In the Classification of a MODU, the environmental actions producing load effects (such as wave, current and wind) that are used in design are selected by the Owner and become a basis of the unit's Classification. It is an assumption of Classification that the Owner will not operate the unit in environmental and other conditions that produce effects that are worse than those reviewed for Classification. This principle carries over to the dynamic response assessment.

In Classification, it is usual that the design storm is expressed deterministically, via the parameters ($H_{\max}$, T_{ass}). However, procedures used to explicitly compute dynamic response mostly rely on a spectral representation of the design-level sea states, so guidance is provided below in Subsection 2/3 on characterizing the design storm sea state in terms of (H_s, T_p) and the defining spectral formulation.

Also in Classification, the *MODU Rules* have specified that the bottoms of the legs should be assumed to penetrate to a depth of at least 3 meters below the seabed, and that each leg end (i.e., spudcan) is pinned (i.e., free to rotate about the axes normal to the leg's longitudinal axes, but fixed against displacements). Recently (2003), a change has been introduced in the *MODU Rules* that affects this practice. When the Owner wishes to credit spudcan-soil rotational stiffness at the bottom of each leg, this can be done in a manner as outlined in Subsection 2/5 below.

3 Spectral Characterization of Wave Data for Dynamic Analysis

The wave conditions provided by the designers for Classification are normally regular waves. The deterministically defined parameters ($H_{\max}$, T_{ass}) of these regular waves need to be converted into spectral parameters (H_s, T_p) to generate the random waves that will be employed in the dynamic analysis.

The following procedures should be used to convert the regular waves:

$$H_{srp} = H_{\max}/1.60 \quad (2.1)$$

$$H_s = (1.0 + 0.5 \times e^{(-d/25)}) \times H_{srp} \quad (2.2)$$

$$T_p = 5.0 \times H_s^{0.5} \quad (2.3)$$

where

H_{srp} = significant wave height, in meters, of the three-hour storm for the assessment return period

H_s = effective significant wave height, in meters

d = water depth, in meters (d > 25 m)

T_p = peak period associated with H_{srp} (also used with H_s), in seconds

Equation (2.2) is the correction for the Wheeler stretching, which counts some nonlinear effects around the free surface. The Pierson-Moskowitz spectrum with the above calculated H_s and T_p is to be used to represent the wave energy. The short-crestedness of waves should not be considered.

5 Spudcan-Soil Rotational Stiffness (SC-S RS)

A 2003 Rule change to the ABS *MODU Rules* permits consideration of "spudcan-soil rotational stiffness" for cases involving dynamic response. The maximum extent to which this rotational stiffness can be applied to the system, $K_{rs,fixed}$, is defined by the following equation.

$$K_{rs,fixed} = E\,I/(L\,C_{\min})$$

where

E = Young's modulus, 209GPa for steel

I = moment of inertia, in m^4

L = the sum of the distance, in m, from the underside of the hull to seabed plus the seabed penetration (minimum 3 meters) $\geq 4.35(I/A_s)^{0.5}$

$C_{\min}$ = $(1.5 - J)/(J + F)$

J = $1 + [7.8\,I/(A_s\,L^2)]$

F = $12\,I\,F_g/(A\,Y^2)$

A = axial area of the equivalent leg, in m^2

A_s = shear area of the leg, in m^2

Y = the distance, in m, between the centerline of one leg and a line joining the centers of the other two legs for a 3-leg unit; the distance, in m, between the centers of leeward and windward rows of legs; in the direction of being considered

F_g = 1.125 for a three leg unit and 1.0 for a four leg unit

The Owner may select values of SC-S RS ranging from zero (the pinned ended condition) up to the maximum value indicated.

SECTION 3 Dynamic Analysis Modeling

1 Introduction

To determine a DAF, a simplified Dynamic Analysis model, as indicated below, may be used. The usual level of modeling employed in this case is designated as an "equivalent model". Inaccurate or inappropriate modeling can have a major impact on the calculated structural responses, therefore, special care should be exercised to assure that the modeling and application of the dynamic loading is done correctly. The stiffness of the Dynamic Analysis model should also be consistent with that of the separate Quasi-Static structural model used to check the adequacy of the structure by the permissible stress unity check criteria of the *MODU Rules*.

3 Stiffness Modeling

The more significant local contributors to overall structural stiffness should be handled with special care. These include:

- Leg stiffness
- Leg-to-hull connection (stiffness of jacking system, proper load transfer direction of guides, pinions and clamps, etc.)
- Leg-to-seabed connection

3.1 Leg Stiffness

The stiffness of a leg is characterized by the following equivalent cross sectional properties:

- Cross sectional area
- Moment of inertia
- Shear area
- Torsional moment of inertia

The dominant factor affecting the system stiffness is leg bending, but other compliance should be incorporated, such as the shear deflection of legs. The shear deflection of most members is small, but it can be significant in a braced structure. Therefore, shear deflection of legs should be properly incorporated in the analysis model.

In an equivalent model, a leg can be modeled by a series of collinear beams. The cross sectional properties of the beams may be derived by employing the formulas given in Appendix 1 or by applying various unit loads to the detailed leg model, if available. If the properties are calculated with the formulas, they may change along the axis of the leg because the properties of the members constituting the leg may vary along the axis of the leg. Although it is not required to model each bay of the leg with a beam element, doing this will facilitate a more accurate mass distribution along the leg.

A spudcan may usually be modeled as a rigid member.

3.3 Hull Stiffness

Hull structure can be modeled as a grillage of beam members. The properties of the beam may be calculated based on the depth of the bulkheads and side shell and the effective width of deck and bottom plating.

The overall structural stiffness or, in turn, the natural period of a unit is less sensitive to hull stiffness. Therefore, the grillage of beams can simply consist of five beam members, or more complicated, with several beam members at each location of the bulkheads and side shell. When considering the contribution from deck(s) and bottom plating, the effective width of deck(s) or bottom plating assigned to a beam member is so determined that the overlapping plan area reaches minimum, i.e., to minimize the areas whose contribution is either not included or included twice. This overlapping will happen when the axes of adjacent beams are not parallel to each other.

The second moment of area of the hull is normally much higher than that of the leg. A common error is to not make the rotational stiffness and "in plane" bending stiffness of equivalent hull members high enough.

3.5 Leg-to-Hull Connection

The leg-to-hull connection is very important to the dynamic analysis. The compliance of the connection is due to a number of factors:

- There may be a global rotation of the leg between the guides due to compliance of the jacking/holding system.
- There may be a global rotation of the leg between the guides due to the local deflection of the guide structure.
- Local deflection of the leg chords, induced by the guide reactions, may lead to an effective rotation of the leg. Also, deformation of the chord wall itself will allow additional leg rotation.

Due to this compliance of the connection, the rotational, horizontal and vertical stiffness of the connection should be modeled with adequate accuracy. A rigid connection is usually not considered as acceptable unless the justification of this simplification is provided.

In an equivalent model, the rotational stiffness of the connection may be represented by rotational springs and horizontal and vertical stiffness by linear springs. The stiffness of the springs may be derived by employing the formulas given in Reference 1 or by applying various load sets to the detailed leg-to-hull connection model, provided the detailed model properly simulates the stiffness of the connection.

3.7 P-Delta Effect

The actual structure will be less stiff than estimated from a linear analysis because of displacement dependent effects, P-δ or Euler amplification. This will tend to increase the deflection of the structure, thereby reducing its effective stiffness. Therefore, the P-δ effect should be accounted for in the Dynamic Analysis model.

A common way to account for the P-δ effect is the geometric stiffness method. In this method, negative stiffness correction terms are introduced into the global stiffness matrix of the Dynamic Analysis model. In order to do this, springs of negative stiffness are connected between each spring's fixed reaction point and a point on each leg where the hull intersects the leg. The negative stiffness for horizontal displacements is given by:

$$K_{pd} = -P_g/L$$

where

P_g = total effective gravity load on each leg, including hull weight and weight of the leg above the hull and leg joint point

L = distance from the spudcan reaction point to the hull vertical center of gravity

3.9 Foundation Stiffness

Additional stiffness to represent the Spudcan-Soil Rotational Stiffness may be included in the model to the extent indicated in Section 5.

One way to implement this in the equivalent model is for each leg to have a spring of specified stiffness connected to the reaction point on the leg and an "earth" point where all degrees of freedom are fixed.

5 Modeling the Mass

The mass that will be dynamically excited and the distribution of that mass should be represented accurately in the Dynamic Analysis model. Items that should be considered include:

- The elevated mass (arising from hull self-weight; mass of additional equipment, variable mass from drilling equipment and consumables and other supplies)
- Leg mass, added mass and any entrained and entrapped (water) mass
- Spudcan mass and entrapped (water) mass

Usually, no mass from functional loads will need to be considered as participating in the dynamic response.

Leg mass can be modeled as nodal mass along the legs. A mass for each bay is adequate for the dynamic analysis. Added mass and any entrained/entrapped mass should be included. If more accurate information about mass distribution is not available, elevated weight may be modeled as nodal masses acting on the hull at its connection to legs.

Generally, the mass distribution of an SEDU, when assessing overall responses, is dominated by the mass of the hull and its associated equipment. The contribution of the legs below the hull is usually not great. But for units with free flooding legs, it is important to include both the entrained water and the added mass.

7 Hydrodynamic Loading

The hydrodynamic loads to be considered in the dynamic analysis are those induced by waves and waves acting with current. The basis of the hydrodynamic loading is Morison's equation, as applied to the Dynamic Analysis model. Equivalent drag and mass coefficients should be developed for the "equivalent leg" idealization of the leg, and as applicable, the spudcan, etc. The current profile should be as specified for Classification, with stretching and compression effects as specified in Reference 1. The hydrodynamic load formulation should consider the relative velocities between the wave and the structure.

When deriving the hydrodynamic properties, such as equivalent diameter, area, drag and mass coefficients of a leg, it is important to account for all members, such as chords, horizontal members, diagonal members, span breakers, etc., in a bay of the leg and their orientations. Some of the properties, i.e., drag coefficient, are storm-heading-dependent.

Where the dynamic analysis is performed considering sea state simulation using random wave generation procedures, as described in Section 4, Airy wave theory can be used to develop the hydrodynamic forces.

9 Damping

Damping can have a significant impact on the response. The total damping ratio to be used in the dynamic response analysis (expressed as a percentage of the critical damping) is defined as:

$$\zeta = c/c_{cr} \cdot 100 \quad \%$$

where

c	=	system damping
c_{cr}	=	critical damping = $2\sqrt{m \cdot k}$
m	=	effective mass of the system
k	=	effective stiffness of the system

The damping ratio should not be taken more than 4%. The three main sources of damping are:

- Structural, including holding system, normally taken as 2% maximum on an independent leg SEDU.
- Foundation, normally taken as 2% maximum for an SEDU with independent legs.
- Hydrodynamic, but since the relative velocity term should be incorporated into either a time domain or frequency domain dynamic analysis, additional damping to account for hydrodynamic damping should not be considered.

SECTION 4 Dynamic Response Analysis Methods

1 General

Fully detailed random dynamic analysis in the time domain or frequency domain may be pursued to obtain dynamic response. However, the "inertial load set" approach is most often used in practical design, and yields sufficiently good results in normal circumstances. In this approach, the dynamic analysis is performed only for determining appropriate values for DAFs. There are several recognized methods to calculate dynamic response. They range from the very simple to the more complex, and the recognized methods can give suitable results in the correct circumstances.

There are three basic approaches. The simplest is referred to as the *Single Degree of Freedom* (SDOF) Approach. The second and third approaches are collectively based on considering the wave (sea-state) as a random quantity. Then, separate calculation methodologies are established based on whether the dynamic response is solved in the time or frequency domain. Due to the limitations of the single-degree-of-freedom (SDOF) approach and the frequency domain approach (refer to Subsection 4/5) the random-wave-time-domain approach is the preferred one to be applied in dynamic response analysis, which will be described in detail in the following Subsection.

Using either a frequency or time domain approach, the most probable maximum extreme (MPME) value is obtained. The MPME is the mode, or highest point, of the *probability density function* (PDF) for the extreme of the response being considered. This is a value with an approximately 63% chance of exceedance, corresponding to the 1/1000 highest peak level in a sea-state with a 3-hour duration. There are a number of methods to predict this extreme response, as will be addressed later in this Section.

3 Time Domain Analysis

3.1 General

The equivalent model indicated in Section 3 is usually employed in time domain analysis. In time domain simulation, a Gaussian random sea state is generated, and the time-step for the simulation is required to be sufficiently small. The duration of the simulation(s) should also be sufficiently long for the method being used to reliably determine the extreme values of the responses being sought.

The overall methodology is to determine the Most Probable Maximum Extreme (MPME) values of the dynamic and static responses in the time domain. The ratio of these two values – defined as DAF – represents the ratio by which the Quasi-Static response, obtained using a high order wave theory and the maximum wave height, should be increased in order to account for dynamic effects. A DAF can be calculated for each individual global response parameter, e.g., base shear, overturning moment or hull sway. Usually, DAF of overturning moment is higher than the other two.

3.3 Random Wave Generation

The wave elevation may be modeled as a linear random superposition of regular wave components, using information from the wave spectrum. The statistics of the underlying random process are Gaussian and fully known theoretically. An empirical modification around the free surface may be needed to account for free surface effects (Wheeler stretching, Equation 2.2). The following criteria are to be satisfied for the generated random waves.

3.3.1 Wave Components

The random wave generation should use at least 200 wave components with divisions of equal wave energy. It is recommended that smaller energy divisions be used in high frequency regions of the spectrum, where the enforcement and cancellation frequencies are located.

3.3.2 Validity of Generated Sea State

The generated random sea state must be Gaussian and should be checked for validity, as follows:

- Correct mean wave elevation
- Standard deviation = $(H_s/4) \pm 1\%$
- $-0.03 < \text{skewness} < 0.03$
- $2.9 < \text{kurtosis} < 3.1$
- Maximum crest elevation = $(H_s/4)\sqrt{2\ln(N)}$ (error within -5% to +7.5%),

where

N = number of wave cycles in the time series being qualified, $N \approx \textit{Simulation Duration}/T_z$

T_z = zero up-crossing period of the wave

3.3.3 Random Seed Effect

Depending on the method used to predict extreme responses and DAF, the random seed effect can be significant. Care should be taken to ensure that the predicted results are not affected by the selection of random seeds.

3.5 Calculation of Structural Response

The structural response should be obtained using the Dynamic Analysis model discussed in Section 3. The analysis model, i.e., the equivalent model with proper loading and boundary conditions, is to be solved using a reliable solver having the capability to do time domain calculations and response statistics calculations. Special attention is to be paid to the topics listed below.

3.5.1 Validity of the Natural Periods of Equivalent Model

The natural periods of a structure are the most important indicators of the dynamic characteristics of the structure. If the computed natural periods are not reasonable, there must be something wrong with the established equivalent model, either its stiffness distribution or its mass distribution, or both. Therefore, the check of natural periods is an indispensable step in the dynamic analysis.

The natural periods of the established equivalent model can be found by solving the eigen-value problem, and the fundamental natural period should be checked against that estimated from the SDOF approach in 4/5.1.

3.5.2 Number of Simulations and Simulation Duration

There are four prevalent methods, as listed in 4/3.7, which can be used to establish the needed MPME values of the response from the time domain analysis. Each of these extreme value prediction methods has specific needs regarding the recommended number and duration of the simulations that should be performed to establish a sufficient statistical basis on which to obtain the MPME value. Therefore, the recommended number and duration of the simulations given below should be followed in the calculation of structural response.

i) *Drag-Inertia Parameter Method*: Simulation time of at least 60 minutes; four simulations with different control parameters, i.e., fully dynamic, quasi-static, quasi-static with C_d (drag coefficient) = 0 and quasi-static with C_{Me} (inertia coefficient) = 0.

ii) *Weibull Fitting method*: Simulation time of at least 60 minutes; number of simulation ≥ 5.

iii) *Gumbel Fitting method*: Simulation time of at least 180 minutes; number of simulation ≥ 10.

iv) *Winterstein/Jensen method*: Simulation time of at least 180 minutes; number of simulation = 1.

More detailed descriptions of these four methods are provided in 4/3.7.

It should be noted that the "quasi-static response analysis" described here and in 4/3.7.1 is performed using the Dynamic Analysis model, but with the mass and damping terms set to zero. This analysis is performed to establish DAFs. It should not be confused with the analysis that is described later with the more detailed, Quasi-Static model that is used to obtain the "unity- checks", as described in Section 5.

3.5.3 Time Step of the Simulations

The integration time-step should be less than, or equal to, the smaller of the following equations, unless it can be shown that a larger time-step leads to no significant change in results.

$T_z/20$ or $T_n/20$

where

T_z = zero up-crossing period of the wave

= $T_p/1.406$ for the Pierson-Moskowitz spectrum

T_n = first mode natural period of the SEDU

3.5.4 Transients

Transient response is to be discarded by removing the first 100 seconds of the response time series before predicting the extreme responses.

3.5.5 Relative Velocity

It is expected that the relative velocity between the wave particle and structural velocities will be included in the hydrodynamic force formulations used in the time domain analysis.

3.7 Prediction of Extreme Responses

Although the waves are considered linear and statistically Gaussian, the structural response of an SEDU is likely to be non-Gaussian due to non-linear drag force and free surface effects which are included in the wave kinematics calculations. The statistics of such a non-Gaussian process are generally not known theoretically, but the extremes are generally larger than the extremes of a corresponding Gaussian random process. For a detailed investigation of the dynamic behavior of an SEDU, the non-Gaussian effects should be included. The four prevalent methods elaborated below are considered acceptable for this purpose.

3.7.1 Method I – Drag/Inertia Parameter Method

The drag/inertia parameter method is based on the assumption that the extreme value of a standardized process can be calculated by splitting the process into two parts, evaluating the extreme values of each and the correlation coefficient between the two, then combining as:

$$(mpm_R)^2 = (mpm_{R1})^2 + (mpm_{R2})^2 + 2\rho_{R12}(mpm_{R1})(mpm_{R2}) \qquad (4.1)$$

The extreme values of the dynamic response can therefore be estimated from the extreme values of the quasi-static response, which is obtained by solving the dynamic equation with both mass term and damping term equal to zero, and the so-called "inertia" response, which is in fact the difference between the dynamic response and the quasi-static response. The correlation coefficient of the quasi-static and "inertia" responses is calculated as:

$$\rho_R = \frac{\sigma_{Rd}^2 - \sigma_{Rs}^2 - \sigma_{Ri}^2}{2\sigma_{Rs}\sigma_{Ri}} \qquad (4.2)$$

The extreme value of the "inertia" response can be reasonably expressed as:

$$mpm_{Ri} = 3.7\ \sigma_{Ri} \qquad (4.3)$$

Reference 1 recommends that the extreme value of the quasi-static response be calculated using one of the three approaches, as follows:

Approach 1: Static extreme can be estimated by combining the extreme of quasi-static response to the drag term of Morison's equation and the extreme of quasi-static response to the inertia term of Morison's equation, using Equation (4.1), above. It is also assumed that the quasi-static responses to the two terms are fully uncorrelated, and hence, the correlation coefficient $\rho_{R12} = 0$ when applying Equation (4.1).

Approach 2: Static extreme may be estimated by using a non-Gaussian measure. The structural responses are nonlinear and non-Gaussian. The degree of non-linearity and the deviation from a Gaussian process may be measured by the so-called drag-inertia parameter, *K*, which is a function of the member hydrodynamic properties and sea state. This parameter is defined as the ratio of the drag force to inertia force acting on a structural member of unit length.

$$K = (2C_D\sigma_V^2)/(\pi C_M D\sigma_A) \qquad (4.4)$$

As an engineering postulate, the probability density function of force per unit length may be used to predict other structural responses by obtaining an appropriate value of *K* from time-domain simulations. *K* can be estimated from the standard deviation of response due to drag force only and inertia force only.

$$K = \sqrt{\frac{\pi}{8}}\,\frac{\sigma_R(C_M = 0)}{\sigma_R(C_D = 0)} \qquad (4.5)$$

Approach 3: Alternatively, K can be estimated from the kurtosis of structural response.

$$K = \left[\frac{(\kappa-3) + \left\{ \frac{26(\kappa-3)}{3} \right\}^{1/2}}{(35-3\kappa)} \right]^{1/2} \qquad (4.6)$$

3.7.2 Method II – Weibull Fitting

Weibull fitting is based on the assumption that for a drag dominated structure, the cumulative distribution of the maxima of the structural response can be fitted to a Weibull class of distribution:

$$F_R = 1 - \exp\left[-\left(\frac{R-\gamma}{\alpha} \right)^{\beta} \right] \qquad (4.7)$$

The extreme value for a specified exceedance probability (e.g., $1/N$) can therefore be calculated as:

$$R = \gamma + \alpha[-\ln(1 - F_R)]^{1/\beta} \qquad (4.8)$$

Using a uniform level of exceedance probability of $1/N$, Equation (4.8) leads to

$$R_{MPME} = \gamma + \alpha[-\ln(1/N)]^{1/\beta} \qquad (4.9)$$

The key issue for using this method is therefore to calculate the parameters α, β and γ, which regression analysis, maximum likelihood estimation or static moment fitting can estimate. For a 3-hour storm simulation, N is approximately 1000. The time series record is first standardized ($R^* = (R - \mu)/\sigma$), and all positive peaks are then sorted in ascending order.

As recommended in Reference 1, only peaks corresponding to a probability of non-exceedance greater than 0.2 are to be used in the curve fitting, and least square regression analysis is used for estimating Weibull parameters.

3.7.3 Method III – Gumbel Fitting

The Gumbel fitting method is based on the assumption that the three-hour extreme values follow the Gumbel distribution:

$$F(x_{extreme} \leq X_{MPME}) = \exp\left[-\exp\left(-\frac{1}{\kappa}(X_{MPME} - \psi) \right) \right] \qquad (4.11)$$

The most probable maximum extreme discussed here corresponds to an exceedance probability of 1/1000 in a distribution function of individual peaks or to 0.63 in an extreme probability distribution function. The MPME of the response can therefore be calculated as:

$$X_{MPME} = \psi - \kappa \ln\{-\ln[F(X_{MPME})]\}$$

$$= \psi - \kappa \ln[-\ln(0.37)] \approx \psi \qquad (4.12)$$

Now, the key issue is to estimate the parameters ψ and κ based on the response obtained from time-domain simulations. Reference 1 recommends that the maximum simulated value be extracted for each of the ten 3-hour response simulations, and that the parameters be computed by maximum likelihood estimation. Similar calculations should also be performed using the ten 3-hour minimum values. Although it is always possible to apply the maximum likelihood fit numerically, the method of moments may be preferred.

For the Gumbel distribution, the mean and variance are given by

Mean: $\mu = \psi + \gamma \cdot \kappa$, γ = Euler constant (0.5772…)

Variance: $\sigma^2 = \pi^2 \kappa^2 / 6$

By which means, the parameters ψ and κ can be directly obtained using the moment fitting method:

$$\kappa = \frac{\sqrt{6}\sigma}{\pi}, \quad \psi = \mu - 0.57722\kappa \qquad (4.13)$$

3.7.4 Method IV – Winterstein/Jensen Method

The basic premise of Winterstein/Jensen method is that a non-Gaussian process can be expressed as a polynomial (e.g., a power series or an orthogonal polynomial) of a zero mean, narrow-banded Gaussian process (represented here by the symbol U), that is

$$R(U) = C_0 + C_1 U + C_2 U^2 + C_3 U^3 \qquad (4.14)$$

The same relationship exists between the MPMEs of the two processes. Since the MPME of Gaussian process U is theoretically known, the MPME of the non-Gaussian process can be calculated if the coefficients C_0, C_1, C_2, C_3 are determined.

3.7.4(a) Determination of U_m. Calculate the following statistical quantities of the time series for the response parameter R under consideration:

μ_R = mean of the process

σ_R = standard deviation

α_3 = skewness

α_4 = kurtosis

Then construct a standardized response process, $z = (R - \mu_R)/\sigma_R$. Using this standardized process, calculate the number of zero-upcrossings, N. In lieu of an actual cycle count from the simulated time series, $N = 1000$ may be assumed for a 3-hour simulation.

The most probable value, U_m, of the transformed process is computed by the following equation:

$$U_m = \sqrt{2\log_e\left(N \cdot \frac{3\text{ hours}}{\text{simulation time (in hours)}}\right)} \qquad (4.15)$$

where U_m is the most probable value of a Gaussian process of zero mean, unit variance.

3.7.4(b) Determination of C coefficients Using Equation (4.14), one can establish the following equations for C_1, C_2 and C_3:

$$\sigma_R^2 = C_1^2 + 6C_1C_3 + 2C_2^2 + 15C_3^2$$

$$\sigma_R^3\alpha_3 = C_2(6C_1^2 + 8C_2^2 + 72C_1C_3 + 270C_3^2)$$

$$\sigma_R^4\alpha_4 = 60C_2^4 + 3C_1^4 + 10395C_3^4 + 60C_1^2C_2^2 + 4500C_2^2C_3^2 + 630C_1^2C_3^2 + 936C_1C_2^2C_3 + 3780C_1C_3^3 + 60C_1^3C_3$$

Solve the equations with the initial guesses as:

$$C_1 = \sigma_R K(1 - 3h_4)$$

$$C_2 = \sigma_R K h_3$$

$$C_3 = \sigma_R K h_4$$

where

$$h_3 = \alpha_3 /[4 + 2\sqrt{\{1 + 1.5(\alpha_4 - 3)\}}]$$

$$h_4 = \left\lfloor \sqrt{\{1 + 1.5(\alpha_4 - 3)\}} - 1 \right\rfloor /18$$

$$K = [1 + 2h_3^2 + 6h_4^2]^{-1/2}$$

Obtain

$$C_0 = \mu_R - \sigma_R K h_3$$

3.7.4(b) Determination of R_{MPME}. The most probable maximum extreme in a 3-hour storm, for the response under consideration, can be computed from the following equation:

$$R_{MPME} = C_0 + C_1 U_m^{\,1} + C_2 U_m^{\,2} + C_3 U_m^{\,3} \quad \text{(4.16)}$$

5 Other Dynamic Analysis Methods

The random wave time domain method is the recommended approach for the dynamic analysis of an SEDU. However, the analysis procedure is relatively complicated and under some circumstances, other methods can also generate results of sufficient accuracy. Besides, some results obtained from the simpler methods, e.g., natural period of the structure determined by SDOF approach, can be used to check the results of time domain analysis. For these reasons, the single degree of freedom approach and frequency domain method are briefly discussed below**.**

5.1 Single-Degree-of-Freedom Approach

In a single-degree-of-freedom (SDOF) approach, the SEDU is modeled as a simple mass/spring/damper system. Due to its simplicity, this approach is recommended for an initial evaluation of the dynamic amplification. If the dynamic amplification determined with this approach is relatively small (i.e., DAF < 1.05), then random dynamic analysis is not required.

5.1.1 Natural Period

The natural period of an SEDU is an important indicator of the degree of dynamic response to be expected. The first and second vibratory modes are usually surge and sway (i.e. lateral displacements at the deck level). The natural periods of these two modes are usually close to each other. Which of the two is higher depends on which direction of the structure is less stiff. The third vibratory mode is normally a torsional mode. Since the period varies with the environmental load direction, care should be taken that the period used in analysis is consistent with the environmental load being considered.

An estimate of the first mode (fundamental) natural period, T_n, is obtained for a single-degree-of-freedom (SDOF) system, as follows:

$$T_n = \frac{1}{f} = 2\pi\sqrt{\frac{M_e}{K_e}}$$

where

f = natural frequency

M_e = effective mass associated with one leg

K_e = effective stiffness associated with one leg, which suitably accounts for the bending, shear and axial stiffness of each leg, the stiffness of the hull-to-leg connection and the degree of spudcan-soil rotational restraint that is to be considered

The detailed information for the calculations of M_e and K_e can be found in Reference 1.

5.1.2 Calculation of the SDOF DAF

The Dynamic Amplification Factor (DAF) of a SDOF system under the influence of a sinusoidal (monotonic) forcing function is given by the following formula:

$$DAF = \frac{1}{\sqrt{\left(1-\Omega^2\right)^2 + \left(2\zeta\Omega\right)^2}}$$

where

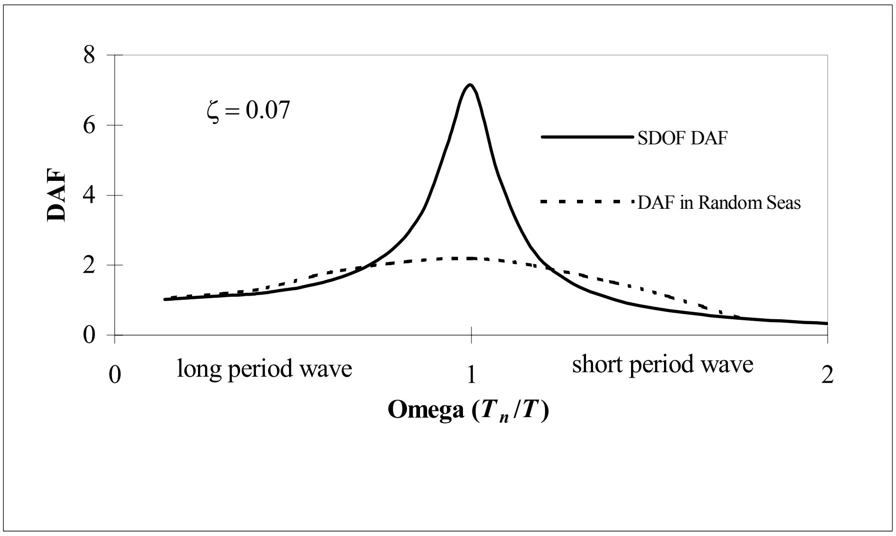
Ω = $\frac{T_n}{T} = \frac{\text{Natural period of the jackup}}{\text{Period of the applied load (wave period)}}$

T = $0.9\ T_p$

ζ = damping ratio

As illustrated in Section 4, Figure 1, if the natural period of the SEDU is equal to the period of the applied load (i.e., Ω is equal to 1.0), the DAF becomes just over 7 (when a damping ratio of 7% is used). Conversely, if there is a very large separation between the natural period and the load period, the DAF could be underestimated. An actual sea state can have a significant spread of energy over the period range, and the curve of DAF against Ω is likely to be much more shallow than that predicted by the SDOF model. This is also illustrated in Section 4, Figure 1.

FIGURE 1
Dynamic Amplification Factor (SDOF)



Care should be taken when determining the appropriate wave period to be used in an SDOF analysis. A range of wave periods should be investigated, along with a range of associated wave heights. The applicable sea states that result in maximum responses should be identified and used in the assessment of the adequacy of the structure's strength.

5.1.3 Dynamic Load Application

The dynamic effect can be applied to the Quasi-Static model by applying an extra force representing the dynamically-induced inertial load set at the center of gravity of the hull structure. The procedure is presented in Section 5.

5.1.4 Limitations

The greatest problems with the SDOF approach are that it may grossly over-estimate the response when the natural period of the unit is close to the monotonic period of the applied load and may possibly underestimate the response when there are large differences in periods. However, this method can give reasonable results when Ω is not too close to unity (outside the range of 0.6 to ~1.3).

5.3 Frequency Domain Dynamic Analysis

A frequency domain analysis combines a spectral representation of the wave energy with a transfer function that defines how the SEDU responds to unit wave heights of different periods, and arrives at the statistical responses in an irregular sea state. In the mathematics of "combining", it is inherently assumed that the wave force (or moment, etc.) on the unit is monotonic at each wave period, and that the responses are linear with wave height. Special care is then needed to determine the appropriate transfer functions to use, and how to produce reasonable results for what is usually a non-monotonic and non-linear loading.

When a frequency domain analysis is pursued, it should be demonstrated that the potential errors arising from the following items are suitably accounted for.

- The wave forces are evaluated using linear kinematics up to the mean water level only, therefore, the wave forces in wave crests may be underestimated.
- Assuming that the responses are linear with wave height. On a drag dominant unit, the wave forces are more closely proportional to the square of the wave height rather than being linear with wave height.
- Failing to account for the higher frequency harmonics in the wave force due to its non-linearity. For example, the force due to a 12 second wave on a drag dominant structure will have a significant component of force at 6 and 4 seconds.
- Improperly interpreting the result and predicting realistic maximum responses from the mean and standard deviation responses that are produced by the analysis.

Since the statistics, e.g., mean and standard deviation, of a response can be obtained from the frequency domain analysis, Method I – Drag/Inertia Parameter Method, indicated in 4/3.7.1, may be employed to predict the MPME, and therefore, DAF of the response.

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SECTION 5 Application of the Inertial Load Set

1 Random Analysis Approach

Once the DAFs are obtained, from either the time domain method or frequency domain method, for the response quantities of interest, they can be used to establish an inertial load set that is imposed on the detailed model of Quasi-Static structural analysis to simulate the dynamic response. Usually, the DAFs for Overturning Moment (OTM) and Base Shear (BS) force are obtained. The inertial load set is combined with all of the other statically considered loads, such as those from wind, currents, deterministically considered storm wave, weights, functional loads, etc. After the strength analysis of the Quasi-Static model is performed, the permissible stress checks, in accordance with the *MODU Rules* unity check equations, can be performed.

A commonly accepted way that the inertial load set is included in the Quasi-Static model is as a concentrated load applied to the elevated hull structure. This idealization is most suitable for the case where the preponderance of the structural system's total mass is in the hull, which is usually considered to be the case. If it were not the case, the complexity of the inertial load set would increase so that instead of a single concentrated load, the inertial load set should be distributed in accordance with the mass distribution and vibratory mode shapes.

The magnitude of the concentrated load representing the dynamic response from waves (or waves acting with current) in the wave loading direction can be obtained from the following quantities:

d = vertical distance from the base of a leg to a location in the elevated hull structure where the concentrated inertial load is to be imposed.

DAF_{OTM} = dynamic amplification factor for overturning moment obtained from the Dynamic Response analysis using the MPME values for the dynamic and statically considered simulated hydrodynamic loads on the unit

DAF_{BS} = dynamic amplification factor for the base shear force obtained from the Dynamic Response analysis using the MPME values for the dynamic and statically considered simulated hydrodynamic loads on the unit

OTM_{QS} = maximum, deterministic overturning moment from the considered wave (or wave acting with current) on the Quasi-Static structural model before the imposition of the inertial load set.

BS_{QS} = maximum, deterministic shear force from the considered wave (or wave acting with current) on the Quasi-Static structural model before the imposition of the inertial load set.

The magnitude of the concentrated inertial force, F_I, is then found from the following equation:

$$F_I = (DAF_{OTM} - 1)\ OTM_{QS}/d$$

It should also be checked that the increased total base shear force from the imposition of the inertial load set is approximately equal (±5%) to the dynamically amplified base shear force, BS_{QS}^*, from the deterministic, Quasi-Static model. This check is expressed as follows:

The increased base shear from the addition of inertial force F_I is:

$$BS_{QS} + F_I$$

and the dynamically amplified base shear is:

$$BS_{QS}^* = BS_{QS}(DAF_{BS} - 1)$$

then

$$0.95\ BS_{QS}^* < (BS_{QS} + F_I) < 1.05\ BS_{QS}^*$$

If this check is not satisfied, various strategies may be pursued to better balance the dynamic effects on the wave-induced overturning moment and base shear force. These may include judiciously adjusting the distance *d*; distributing the inertial load set according to the mass distribution and vibratory mode shapes, etc. However, when $(BS_{QS} + F_I) > BS_{QS}^*$ and no adjustment is made, the structural assessment will tend to be conservative.

3 SDOF Approach

When the approach presented in 4/5.1 is applied, the procedure that may be followed to establish the inertial load set is as follows.

The magnitude of the force is determined from:

$$F_i = (DAF - 1) \times F_{wave\ amp}$$

where

F_i = inertial load set to be applied at the center of gravity of the hull and associated leg and in the direction of waves (i.e., that contained in, and above, the lower guide)

DAF = SDOF dynamic amplification factor

$F_{wave\ amp}$ = static amplitude wave force = $0.5(F_{\max} - F_{\min})$

$F_{\max}$, $F_{\min}$ = maximum/minimum total combined wave and current force (or wave/current base shear) obtained from quasi-static structural analysis, using the appropriate sea state

5 Evaluation of Structural Adequacy

The inertial load set is combined with all of the other loads that should be included in the detailed Quasi-Static structural analysis model that is used to obtain the stresses and deflections that are evaluated with respect to the acceptance criteria given in the *MODU Rules*.

APPENDIX **1** **Equivalent Section Stiffness Properties of Lattice Legs**

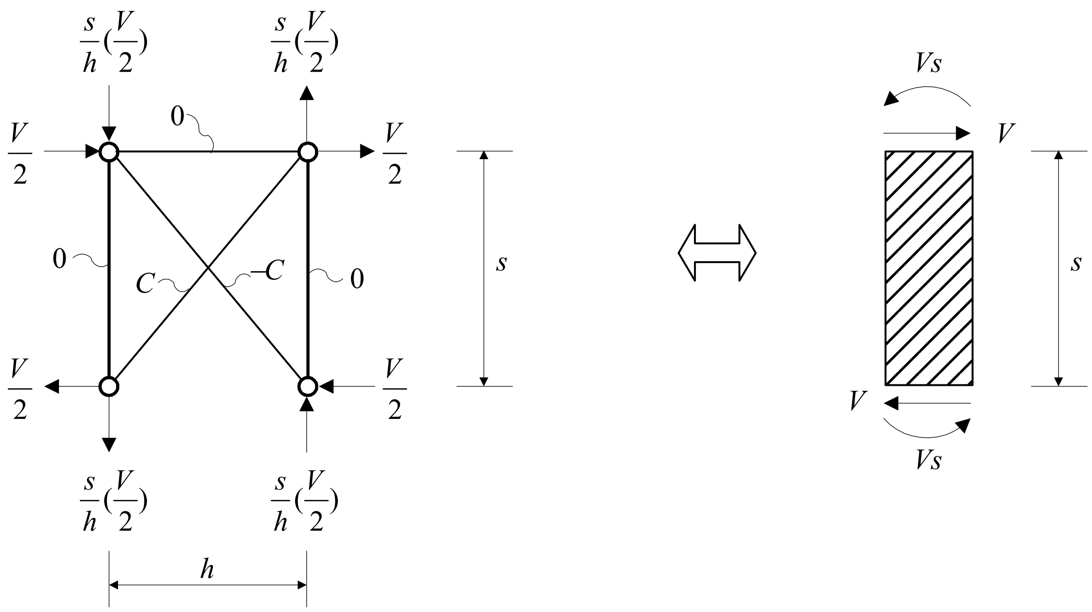
1 Introduction

In order to evaluate the equivalent section stiffness properties of 3D lattice legs, it is necessary first to identify the equivalent shear area of 2D lattice structures, which composes each wall of the 3D lattice legs and the equivalent polar moment of inertia of the 3D lattice leg's cross-section. The equivalent shear area uses the equivalent 2D lattice shear area of the structure.

3 Equivalent Shear Area of 2D Lattice Structures

The equivalent shear area of 2D lattice structures is evaluated by the principle of virtual work, as indicated in Reference 4. For example, Appendix 1, Figure 1 shows that the strain energy of the shear beam deformation is made equivalent to the complimentary virtual work in the X bracing system.

FIGURE 1
Shear Force System for X Bracing and its Equivalent Beam



The forces in the diagonals are $\pm C = \pm(V/2)d/h$, where d is the diagonal length, and the corresponding complementary energy for the 2D lattice truss is:

$$W^* = \frac{1}{2}\sum_i \frac{F_i^2 L_i}{EA_i} = 2\left(\frac{1}{2}\frac{C^2 d}{EA_D}\right) = \frac{1}{4}\frac{V^2 d^3}{h^2 EA_D} \quad \text{(A1.1)}$$

where F_i , L_i, A_i are the force, length and area of the i-th member, and E is the modulus of elasticity. According to the principle of virtual forces, one obtains

$$s\frac{V}{GA_Q} = \frac{\partial W^*}{\partial V} = \frac{1}{2}\frac{Vd^3}{h^2 EA_D} \quad \text{(A1.2)}$$

where

G = $E/[2(1 + \nu)]$

A_Q = equivalent shear area of the structure

Then

$$A_Q = \frac{(1+\nu)h^2 s}{\dfrac{d^3}{4A_D}} \quad \text{(A1.3)}$$

The formulae for four types of 2D lattice structures commonly employed in constructing the legs of an SEDU are derived and listed in Appendix 1, Table 1. The shear areas calculated by these formulae are very close to the results formulae presented in Reference 1 for typical SEDUs in operation.

5 Equivalent Section Stiffness Properties of 3D Lattice Legs

The equivalent section stiffness properties of 3D lattice legs are obtained as follows:

i) The cross-sectional area of a leg is the summation of the cross-sectional areas of all of the chords in the leg. The contribution from the braces is neglected.

ii) The shear area of a leg's cross-section in k direction (i.e., y or z direction) can be expressed as

$$A_{Qk} = \sum_{i=1}^{N} A_Q \sin^2\beta_i$$

where

A_Q = equivalent shear area of 2D lattice structure

β_i = angle between k direction and the normal direction of the i-th 2D lattice structure

N = total number of the 2D lattice structures in the leg (i.e., 3 or 4)

iii) The moment of inertia of the leg's cross-section for k direction (i.e., y or z direction) is the summation of the cross-sectional area of a chord times the square of the distance from the chord center to the neutral axis of the leg's cross-section in k direction for all chords. The contribution from the braces is neglected.

iv) The polar moment of inertia of the leg's cross-section is

$$I_T = \sum_{i=1}^{N} A_Q \ell_i^{\,2}$$

where

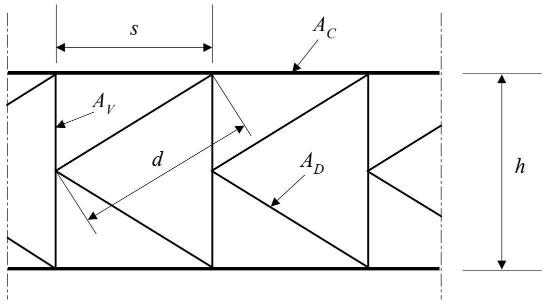
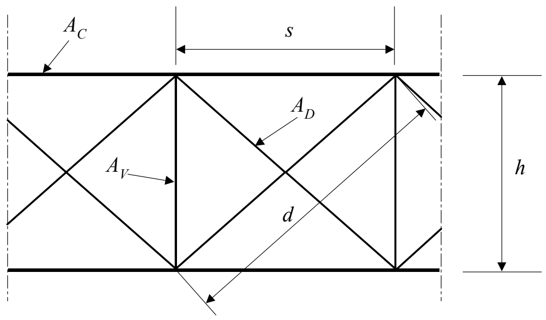
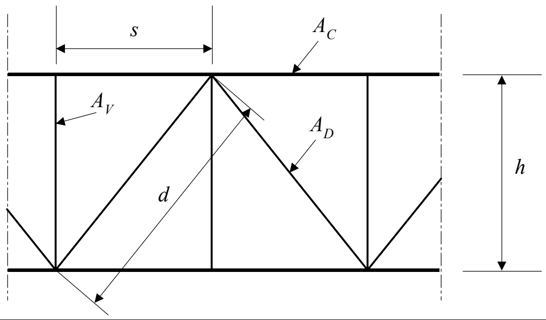
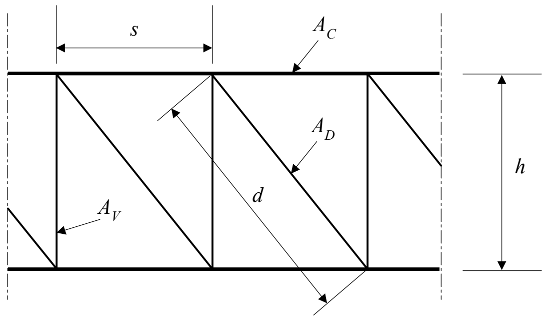
A_Q = equivalent shear area of 2D lattice structure

ℓ_i = distance from the *i*-th 2D lattice structure to the geometry center of the leg's cross-section

N = total number of the 2D lattice structures in the leg (i.e., 3 or 4)

Appendix 1, Table 2 presents the equivalent beam moment of inertial, which when multiplied by the modulus of elasticity provides the section stiffness properties of three types of leg configurations.

TABLE 1
Equivalent Shear Area of 2D Lattice Structures

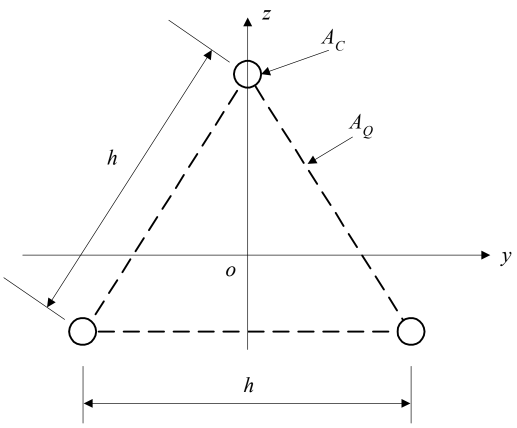
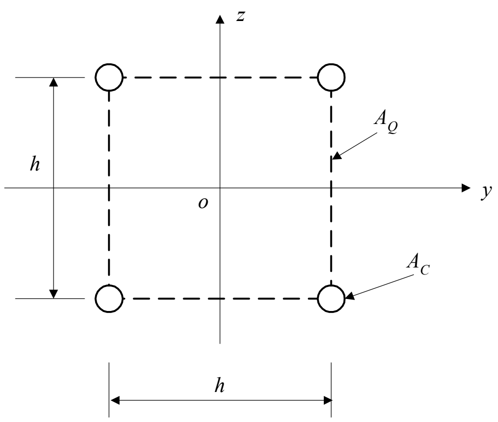
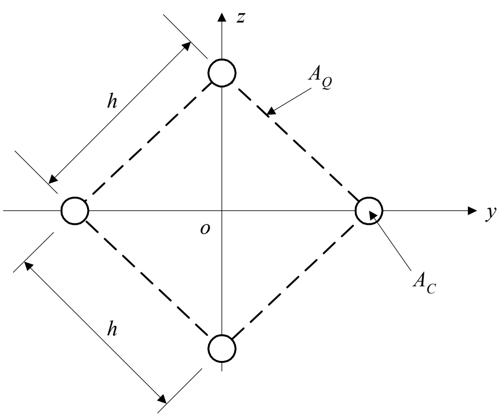
<table>
<tr><th>Structure</th><th>Equivalent Shear Area</th></tr>
<tr><td>
</td><td>$$A_Q = \frac{(1+\nu)h^2 s}{\frac{d^3}{A_D} + \frac{h^3}{8A_V} + \frac{s^3}{4A_C}}$$</td></tr>
<tr><td>
</td><td>$$A_Q = \frac{(1+\nu)h^2 s}{\frac{d^3}{4A_D}}$$</td></tr>
<tr><td>
</td><td>$$A_Q = \frac{(1+\nu)h^2 s}{\frac{d^3}{2A_D} + \frac{s^3}{A_C}}$$</td></tr>
<tr><td>
</td><td>$$A_Q = \frac{(1+\nu)h^2 s}{\frac{d^3}{2A_D} + \frac{h^3}{2A_V} + \frac{s^3}{4A_C}}$$</td></tr>
</table>

Note:

ν = Poisson ratio

A_k = cross sectional area of the corresponding member ($k = C$, D or V)

TABLE 2
Equivalent Moment of Inertia Properties of 3D Lattice Legs

Leg Configuration	*Equivalent Section Stiffness Properties*
	$A = 3A_C$ $A_{Qy} = A_{Qz} = 3A_Q / 2$ $I_y = I_z = A_C h^2 / 2$ $I_T = A_Q h^2 / 4$
	$A = 4A_C$ $A_{Qy} = A_{Qz} = 2A_Q$ $I_y = I_z = A_C h^2$ $I_T = A_Q h^2$
	$A = 4A_C$ $A_{Qy} = A_{Qz} = 2A_Q$ $I_y = I_z = A_C h^2$ $I_T = A_Q h^2$

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APPENDIX 2 References

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3. Dynamics of Structures, Second Edition, R. Clough and J. Penzien, 1993
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5. Continuum Models for Beam and Platelike Lattice Structures, Anne Teughels, IASS-IACM 2000, Fourth International Colloquium on Computation of Shell and Spatial Structures, Chania – Crete, Greece, June 5-7, 2000

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